Nonstationary spectral peak estimation based on Monte Carlo Filter

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Abstract A new method to estimate nonstationary power spectrum that has multiple peaks is proposed. In the method, peaks of power spectrum are varying with time. Smoothness priors are assumed to frequency and bandwidth of the peaks in system model. State vector consists of the frequencies and the bandwidths. Observation model is time-varying coefficient AR model in which the coefficients are nonlinearly parametrized by the state vector. A nonlinear state space representation is formed by the system model and the observation model. As a method to estimate the state of the nonlinear model, Monte Carlo filter proposed by G.Kitagawa is used. Simulational experiments to evaluate the estimation precision of peak frequencies compared with the ordinary model are reported.

Keywords: Nonstationary, Spectral analysis, Monte Carlo

1. Introduction

A method to estimate multiple peaks of a nonstationary power spectrum using Time-Varying Peak frequencies of Power spectrum (TVPP) model is reported. TVPP model has an advantage that can estimate frequency of spectral peaks directly due to its nonlinear formulation based on characteristic roots. However, the conventional research [2] on the model has a significant limitation with respect to the number of peaks, caused by computational complexity of non-Gaussian state esimation method based on numerical approximation [3].

In this paper, the limitation of the number of peaks is lessened with the aid of Monte Carlo Filter(MCF) proposed by G.Kitagawa[4]. The key idea of MCF is an approximation of non-Gaussian distribution by its realizations. Then we can make a reduction of computational complexity from the order of exponential with respect to the number of peaks, which is a cost of the numerical approximation method, to the order of the number of realizations. The efficiency of the model has been demonstrated in simulational experiments, by comparing TVPP model to the conventional model.

2. Model

Time-Varying Peak frequencies of Power spectrum(TVPP) model is defined by a state space representation [2]

$$\mathbf{x}_t = \mathbf{x}_{t-1} + \mathbf{w}_t, \quad (1)$$

$$y_t = \mathbf{a}_t{}' \; \mathbf{y}_t + \varepsilon_t \quad (2)$$

where $'$ denotes transpose. Eq.(1) is system model which represents smoothness of state vector $\mathbf{x}_t$. The smoothness is governed by a system noise vector $\mathbf{w}_t$. Observation model eq.(2) forms time-varying coefficient AR model of order $p = 2m$, where y_t is observation, and ε_t is observation noise that belongs to $N(0, \sigma^2)$. $\mathbf{y}_t$ is a vector of past p observations

$$\mathbf{y}_t = [y_{t-1}, y_{t-2}, \cdots, y_{t-p}]' \quad (3)$$

The p-dimensional vector $\mathbf{a}_t = \mathbf{a}(\mathbf{x}_t)$ consists of time-varying AR coefficients nonlinearly parametrized by the state vector $\mathbf{x}_t$.

Items of the state vector are

$$\mathbf{x}_t = [\theta_{1,t}, \theta_{2,t}, \cdots, \theta_{m,t}, r_{1,t}, r_{2,t}, \cdots, r_{m,t}]' \quad (4)$$

where $\theta_{j,t}$, $j = 1, 2, \cdots, m$ are peak frequencies and $r_{j,t}$, $j = 1, 2, \cdots, m$ are peak bandwidths of power spectrum. Note that $r_{j,t}e^{\pm i\theta_{j,t}}$, $j = 1, 2, \cdots, m$ are roots of characteristic equation

$$1 - a_{1,t}\, s - a_{2,t}\, s^2 \cdots - a_{p,t}\, s^p = 0 \quad (5)$$

where $a_{1,t}, a_{2,t}, \cdots, a_{p,t}$ are items of vector $\mathbf{a}_t$. Although it is difficult to obtain state vector $\mathbf{x}_t$ from AR coefficient vector $\mathbf{a}_t$ since we have to solve eigen value problem, the opposite calculation, obtaining time-varying AR coefficient vector $\mathbf{a}_t$ from state vector $\mathbf{x}_t$, is easily executed.

3. Estimation

Nonlinear non-Gaussian state estimation method should be used to estimate the state

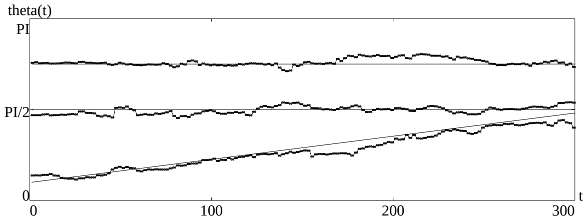

Figure 1: Estimated time-varying peak frequency for 3-peaks-data by TVPP model.

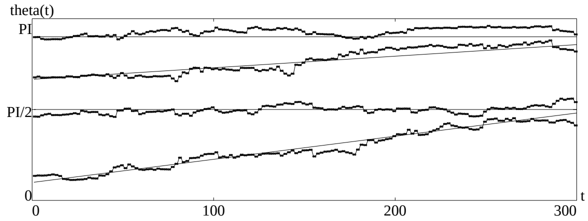

Figure 2: Estimated time-varying peak frequency for 4-peaks-data by TVPP model.

vector because of the nonlinearity of the proposed model. Numerical approximation [3] was used in [2], however the method cannot be applied to high dimensional state vector. Then we have employed Monte Carlo filter [4] instead of numerical approximation [3].

Hyper parameters, variances of system noise τ_θ^2 and τ_r^2, and variance observation noise σ^2 are estimated based on ABIC [1]. Practically, several candidate values of these parameters are tried and obtain ABIC value for all combination of the values, then we can decide the best combination of the candidates. The order of the model, denoted by $p = 2m$, is also determined based on ABIC.

4. Simulation

Two experiments, 3-peaks-data and 4-peaks-data, are demonstrated. Simulation data sets are generated by TVPP model as the true model. Then assume we know none of the true model, the true parameter values, and the true order. Under this assumption, TVPP model has been applied. TVCAR model [5] is applied as an ordinary model to the same data sets. Results by TVPP model are shown in fig.1 and fig.2. Results by TVCAR model are shown in fig.3 and fig.4. For each figure, solid lines show the true peak frequency, and plots show the estimated one. In TVCAR model, the plots are the results of eigen value calculated by QR-decomposition from estimated AR coefficients.

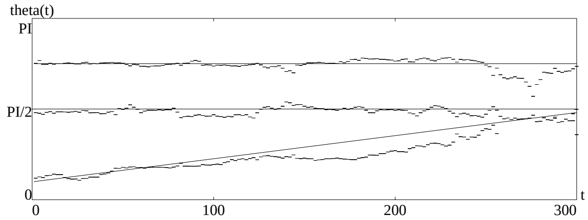

Figure 3: Estimated time-varying peak frequency for 3-peaks-data by TVCAR model.

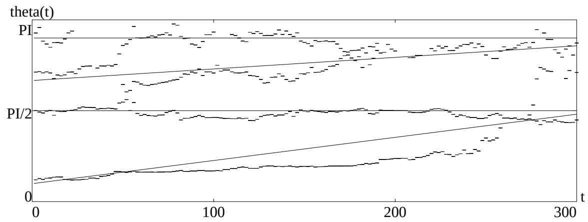

Figure 4: Estimated time-varying peak frequency for 4-peaks-data by TVCAR model.

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