Nonstationary Spectral Peak Estimation by Monte Carlo Filter

Norikazu Ikoma
Kyushu Institute of Technology
Tobata-ku, Kita-Kyushu, 804-8550 Japan
ikoma@comp.kyutech.ac.jp

Hiroshi Maeda
Kyushu Institute of Technology
Tobata-ku, Kita-Kyushu, 804-8550 Japan
hmaeda@comp.kyutech.ac.jp

Abstract

The aim of research is to estimate multiple peaks of power spectrum that are varying with time. A model to estimate the time-varying peaks has been proposed by the author. The model is written in a state space representation composed by a system model and an observation model. The system model denotes smooth change of a state vector that consists of pairs of peak frequency and bandwidth. The observation model is autoregressive model with time-varying coefficients that are nonlinearly parametrized by the state vector. The nonlinear parametrization is based on a fact that the pairs of frequency and bandwidth are roots of characteristic equation of the autoregressive model. Estimating the state vector by giving the observations results the estimation of frequency and bandwidth pairs of time-varying power spectrum. In state estimation, properties of nonlinear and non-Gaussian should be treated because of the nonlinear formulation of the model. As a method of state estimation, we have employed an approximation of non-Gaussian distribution by its realizations, called Monte Carlo filter. Through numerical examples, estimation precision of peak frequency has been checked by comparing with a conventional model.

1. Introduction

Estimation of nonstationary spectrum from time series data is one of the most important topic in the recent researches of random signal analysis. It can be applied to wide area of research and development fields, for example, analysis of seismic wave data, vibration under nonstationary conditions, acoustic signal processing, human voice analysis, speech recognition and so on. Also there are so many techniques for nonstationary spectral analysis by using, for example, covariance and spectral window, or methods based on some specific definition of nonstationary spectrum. Among the many techniques of nonstationary spectrum estimation, there is one effective approach that uses time-varying AutoRegressive(AR) coefficients. In this approach, nonstationary spectrum can be obtained by estimating time-varying AR coefficients from the observation series.

Several models of this approach, using time-varying AR coefficients, have been proposed. Locally stationary AR model [12] uses stationary models for each short interval in which the data can be considered as stationary. Based on the locally stationary AR model, an modification employing smoothness between the adjoining intervals is also proposed [13]. The smoothness used here is accomplished by introducing a prior distribution in the context of Bayesian approach [2], where weighted sum of squares error of prediction, i.e., fitness to the data, and smoothness represented by 1st or 2nd difference appear and trade-off parameter of these two factors is determined by maximizing likelihood. Time-Varying Coefficient AutoRegressive(TVCAR) model [10] also assumes smoothness of this approach to AR coefficient for each time. In TVCAR model, time-smoothness of spectrum is obtained through the assumption of smooth changes of AR coefficients.

These conventional models can be written in linear Gaussian state space representation, so the state, which represents time-varying AR coefficients, can be estimated by using Kalman filter. However, the reason for assuming a smoothness to the AR coefficient is rather mathematical one than a motivation for developing an effective model with deep consideration of the object feature. The underlying fact for this is that linear Gaussian model is easy to handle. Recently, several methods for nonlinear non-Gaussian filtering have been researched. Difference among these methods is the approximation non-Gaussian distribution. Gaussian sum approximation [3] is an early research. Numerical approximation of distribution [7], [8] uses numerical integration. The most recent one is an approximation by realizations(particles) drawn from the non-Gaussian distribution. Some researchers of this approximation independently proposed such as bootstrap filter[4], conditional density propagation(CONDENSATION) [6], and Monte Carlo filter(MCF)[9].

Taking into account the recent researches of non-Gaussian filter, the author has been proposed a model for the purpose to estimate nonstationary power spectrum in which peaks are smoothly changing with time, and has named it Time-Varying Peak frequencies of Power spectrum (TVPP) model [5]. In state space representation of TVPP model, state vector contains peak parameters that consist of pairs of frequency and bandwidth of spectral peaks. Advantages of TVPP model compared with TVCAR model are 1) smoothness is assumed to the spectral peaks directly, 2) peak parameters can be estimated directly. Due to nonlinear factor contained in TVPP model, there is a disadvantage of computational cost since non-Gaussian nonlinear filtering should be applied to the state estimation. Though TVPP model has a disadvantage with respect to computational cost for non-Gaussian filter, it is reasonable to use when the direct formulation of peak parameters is desirable.

As a method of non-Gaussian nonlinear filtering, we have employed Monte Carlo filter(MCF) [9]. The key idea of MCF is an approximation of non-Gaussian distribution, which appears in filtering procedure, by particles drawn from the distribution. Procedure of filtering is written by procedures of each particle. Compared with the numerical approximation method [7], [8], in which computational cost is exponential order with respect to the dimension of state vector, computational cost of non-Gaussian filtering by MCF is lessen to the order of number of particles. In the conventional research [5], only two peaks has been reported due to a limitation of the number of peaks caused by the expensive cost of computation by the numerical approximation method. With the aid of MCF, we can apply much more number of peaks to TVPP model in practice.

In this paper, we will show a method to estimate multiple peak of nonstationary spectrum by TVPP model with MCF [9]. Since the computational cost of MCF is less expensive than that of numerical approximation method [7], [8], the limitation of number of peaks of TVPP model is relaxed with the aid of MCF. We firstly show a formulation of TVPP model based on a state space representation. Secondly, general state space representation and MCF as the state estimation method are summarized. After these formulations, efficiency of the model has been checked by a simulational experiment. Comparison of TVPP model to TVCAR model has been examined with respect to the values of AIC [1] and mean squares error of the estimated peak frequency. Finally, we will make some concluding remarks based on the result of the simulational experiment.

2. Time-Varying Peak frequencies of Power spectrum model

Time-Varying Peak frequencies of Power spectrum(TVPP) model [5] is defined by a nonlinear state space representation

$$\mathbf{x}_t = \mathbf{x}_{t-1} + \mathbf{w}_t, \tag{1}$$

$$y_t = \mathbf{a}(\mathbf{x}_t)' \ \mathbf{y}_t + \varepsilon_t \tag{2}$$

where $'$ denotes transpose. Eq.(1) is called system model which represents smoothness of state vector $\mathbf{x}_t$, where the smoothness is governed by a system noise vector $\mathbf{w}_t$. Eq.(2) is called observation model that forms time-varying coefficient AR model of even order $p = 2m$, where y_t is observation, and ε_t is observation noise of $N(0, \sigma^2)$. $\mathbf{y}_t$ is a vector of past p observations such that $\mathbf{y}_t = [y_{t-1}, y_{t-2}, \cdots, y_{t-p}]'$. $\mathbf{a}_t \equiv \mathbf{a}(\mathbf{x}_t)$ is p-dimensional vector of time-varying AR coefficients that is nonlinearly parametrized by the state vector $\mathbf{x}_t$.

Items of the state vector are as follows;

$$\mathbf{x}_t = [\theta_{1,t}, \theta_{2,t}, \cdots, \theta_{m,t}, r_{1,t}, r_{2,t}, \cdots, r_{m,t}]' \tag{3}$$

where $\theta_{j,t}$ are peak frequencies of power spectrum at time t, and $r_{j,t}$ are bandwidths of corresponding peak for each $j = 1, 2, \cdots, m$. We obtain items of vector $\mathbf{a}_t$, which are denoted by $a_{1,t}, a_{2,t}, \cdots, a_{p,t}$, by

$$\begin{aligned} A(z; \mathbf{x}_t) &= \prod_{k=1}^{m} (1 - r_k^{-1} e^{-i\theta_k} z)(1 - r_k^{-1} e^{i\theta_k} z) \\ &= 1 - a_{1,t}\ z - a_{2,t}\ z^2 - \cdots - a_{p,t}\ z^p. \end{aligned} \tag{4}$$

By considering z as backward shift operator defined by $zy_t = y_{t-1}$, the observation model (2) can be written as

$$A(z; \mathbf{x}_t)\ y_t = \varepsilon_t. \tag{5}$$

Note that $r_{j,t} e^{\pm i\theta_{j,t}}$, $j = 1, 2, \cdots, m$ are roots of characteristic equation $A(z; \mathbf{x}_t) = 0$. This means that the model consists of a sequentially connected system of m number of AR(2)'s, where the input of the system is the observation noise ε_t and the output is y_t. Consequently, transfer function of the model is denoted by inverse of eq.(4), $1/A(z; \mathbf{x}_t)$.

In each AR(2), complex conjugate roots are smoothly varying with time as denoted in system model (1). Although it is difficult to obtain state vector $\mathbf{x}_t$ from AR coefficient vector $\mathbf{a}_t$ since we have to solve eigen value problem, however, calculation of opposite direction shown in eq. (4) is easily executed.

Nonstationary power spectrum(instantaneous power spectrum) of TVPP model can be obtained through a natural generalization of power spectrum of stationary AR model into nonstationary AR model. Since the transfer function of the model is inverse of eq.(4) and the input signal ε_t has spectrum σ^2, nonstationary power spectrum of TVPP model becomes

$$p(\omega, t) = \frac{\sigma^2}{|A(e^{-i\omega}; \mathbf{x}_t)|^2}, \tag{6}$$

where $\omega \in [0, \pi]$ is angular frequency.

3. State estimation by Monte Carlo filter

Most of time series models can be written in general state space representation that consists of system equation and observation equation

$$\begin{aligned} \mathbf{x}_t &= \mathbf{g}(\mathbf{x}_{t-1}, \mathbf{w}_t), & (7)\\ y_t &= h(\mathbf{x}_t, \varepsilon_t). & (8) \end{aligned}$$

In system equation (7), $\mathbf{x}_t$ is a state vector(p-dimension), $\mathbf{w}_t$ is a system noise(l-dimension, $l \leq p$) with distribution $q(\mathbf{w}; \tau)$, where τ is a vector of system noise distribution parameters. $\mathbf{g}(\cdot, \cdot)$ is a function $\mathcal{R}^p \times \mathcal{R}^l \mapsto \mathcal{R}^p$. In observation equation (8), y_t is observation, ε_t is observation noise with distribution $r(\varepsilon; \sigma)$, where σ is a parameter of observation noise distribution. $h(\cdot, \cdot)$ is a function $\mathcal{R}^p \times \mathcal{R} \mapsto \mathcal{R}$, and we assume $\varepsilon_t = h^{-1}(y_t, \mathbf{x}_t)$ exisits. Note that, y_t and ε_t are assumed to be scalar in this paper, however, in general, they are not necessary to be scalar.

State estimation problem is to estimate the distribution of state vector $\mathbf{x}_t$ by giving observations, for each $t = 1, 2, \cdots, N$. When the given observation set is $Y_{t-1} = \{y_1, y_2, \cdots, y_{t-1}\}$ the state distribution is called one-step-ahead prediction, and when $Y_t = \{y_1, y_2, \cdots, y_t\}$ is given the state distribution is called filtering. They are alternatively estimated in order of time index t.

There are several conventional researches of state estimation problem for general state space representation, such as approximation of non-Gaussian distribution by Gaussian-sum [3], numerical approximation of distribution [7], [8], and approximation by realizations(particles) drawn from the non-Gaussian distribution. However, Gaussian sum approximation method remains a problem how to reduce the increasing number of Gaussian distributions. Numerical approximation method has a computational limitation since the cost of calculation is exponential order of the grid number for numerical approximation. The third method, which uses particles for the approximation, is currently the most practical one among them since computational cost is reasonable compared with the numerical approximation method. In fact, computational cost of the particle approximation method is lessen to the order of the number of particles. So we have employed the method using particles. Several similar methods have been proposed independently, such as bootstrap filter[4], conditional density propagation(CONDENSATION) [6], and Monte Carlo filter(MCF)[9]. We have employed the MCF method among them.

The key idea of MCF is an approximation of non-Gaussian distribution by its realization particles. Filtering procedures can be done by using these particles instead of distribution itself. Notation of particles for one-step-ahead prediction is

$$\left\{\mathbf{p}_1^{(t)}, \mathbf{p}_2^{(t)}, \cdots, \mathbf{p}_M^{(t)}\right\} \sim p(\mathbf{x}_t | Y_{t-1}), \tag{9}$$

and that for filtering is

$$\left\{\mathbf{f}_1^{(t)}, \mathbf{f}_2^{(t)}, \cdots, \mathbf{f}_M^{(t)}\right\} \sim p(\mathbf{x}_t | Y_t). \tag{10}$$

By using these particles, estimation of prediction and filtering distributions, which are represented by the corresponding particles, are obtained through the following calculations

$$\mathbf{p}_i^{(t)} = \mathbf{g}(\mathbf{f}_i^{(t-1)}, \mathbf{w}_i^{(t)}) \tag{11}$$

where $\left\{\mathbf{w}_1^{(t)}, \mathbf{w}_2^{(t)}, \cdots, \mathbf{w}_M^{(t)}\right\} \sim q(\mathbf{w}; \tau)$.

Calculate likelihood of each particle by

$$\alpha_i^{(t)} = p(y_t | \mathbf{p}_i^{(t)}) = r\left(h^{-1}(y_t, \mathbf{p}_i^{(t)}); \sigma\right). \tag{12}$$

Resample as

$$\mathbf{f}_i^{(t)} = \begin{cases} \mathbf{p}_1^{(t)} & \text{with probability} \quad \alpha_1^{(t)} / \sum_{j=1}^M \alpha_j^{(t)} \\ \vdots & \qquad \vdots \qquad\qquad\qquad \vdots \\ \mathbf{p}_M^{(t)} & \text{with probability} \quad \alpha_M^{(t)} / \sum_{j=1}^M \alpha_j^{(t)} \end{cases} \tag{13}$$

By starting from realizations of $p(\mathbf{x}_0 | Y_0)$, alternatively applying (11) and (13) according to the order of t, the approximated estimations of $p(\mathbf{x}_t | Y_{t-1})$ and $p(\mathbf{x}_t | Y_t)$ for all $t = 1, 2, \cdots, N$ can be obtained.

After all estimates are calculated, likelihood of the model to the given data set can be approximately obtained by

$$\begin{aligned} l(\vartheta) &= \sum_{t=1}^{N} \log p(y_t | Y_{t-1}) \\ &\simeq \sum_{t=1}^{N} \log \left(\sum_{j=1}^{M} \alpha_j^{(t)} \right) - N \log M, \end{aligned} \tag{14}$$

where, $\vartheta = [\sigma, \tau]$ is called "hyperparameter" [2] that governs the performance of state estimation. The optimal value of hyperparameter, denoted by $\hat{\vartheta}$, is determined by minimizing the log-likelihood (14). Akaike

Information Criterion(AIC) defined by [1];

$$\mathrm{AIC} = -2l(\hat{\vartheta}) + 2(\#\text{ of free parameters}) \qquad (15)$$

is used for order determination and model selection

4. Simulation

In order to evaluate the efficiency of TVPP model, a simulational experiment by using artificially generated data sets has been done. In the experiment, comparison to TVCAR model with respect to AIC and mean squares error of peak frequency has been shown.

4.1 Data

Two data sets, "3-peaks-data" and "4-peaks-data" are generated by simulation. In the simulation, a model based on TVPP model is used as the true model. In the true model, frequencies and bandwidths of power spectrum are denoted by $\theta^*_{k,t}$ and $r^*_{k,t}$ respectively. Some peaks have time-varying frequency with constant increment, and other peaks have time constant frequency. Bandwidths for all peaks are time invariant. The conditions of the simulation for each data set are as follows;

3-peaks-data

$r^*_{1,t} = r^*_{2,t} = r^*_{3,t} = 1/1.2,$
$\theta^*_{1,t} = 0.314 + 0.004t,$
$\theta^*_{2,t} = 1.571, \qquad \theta^*_{3,t} = 2.356$

4-peaks-data

$r^*_{1,t} = 1/1.1,\ r^*_{2,t} = r^*_{3,t} = r^*_{4,t} = 1/1.2,$
$\theta^*_{1,t} = 0.314 + 0.004t, \qquad \theta^*_{2,t} = 1.571,$
$\theta^*_{3,t} = 2.094 + 0.002t, \qquad \theta^*_{4,t} = 2.827$

These conditions are assumed to be unknown in the estimation step, and the purpose of this experiment is to estimate the changes of these variables. The number of peaks for each data set is also unknown and is to be determined in the experiment. Observation noise is $N(0, \sigma^2)$ with $\sigma^2 = 1.0$ in the simulation step, and we assume it is known in the estimation step.

4.2 Estimation

Conditions of MCF for state estimation of TVPP model are explained here in detail. Firstly, number of particles is set to $M = 1000$. This is due to the rule of thumb shown in [9] for the estimation of average of state distribution. There is another factor to be set that is locally stationarity factor. This factor is defined by the rate of prediction with respect to time index t. Theoretically, it is 1, i.e., one-step-ahead prediction and filtering are alternatively done. It can be changed depending on practical reason, and we have set it to 2 in this experiment.

Frequency and bandwidth are in state vector, and their feasible region are $[0, \pi]$ and $(0, 1]$, respectively. Restrictions are required at the calculation of one-step-ahead prediction eq.(11) not to be out of range. Although several implementations of the restrictions can be considered, we have employed one of the simplest way that use the correction of these values when they are out of range. Furthermore, due to practical reason to detect the peak of power spectrum, we have used $[r_c, 1]$ as the region for bandwidth with $r_c = 0.5$.

Pre-filtering is done to obtain the appropriate initial distribution from the information of leading part of the series. It starts from non-informative distribution $p(\mathbf{x}_0|Y_0)$ given by taking peak frequencies at regular interval

$$\theta_{k,0} = \pi k/(m+1), \qquad k = 1, \cdots, m. \qquad (16)$$

and bandwidths constant value $r_{k,0} = r_0$. For the purpose to estimate "peaks" of power spectrum, the value r_0 should be large enough to be identified as "peak", and we have employed $r_0 = 0.8$ in this experiment. We have used $t \leq 20$ of series as the leading part. After finishing the pre-filtering, then going back to the head of the series, we do the filtering procedure starting from the distribution obtained by the pre-filtering.

We have used Gaussian distribution for the system noise $q(\mathbf{w}; \tau)$ with diagonal covariance matrix. By assuming the same variance for all peaks independently to frequency and bandwidth, the diagonal part τ^2 can be identified by two variance parameters τ^2_θ for frequency and τ^2_r for bandwidth. Number of peaks, $m = \{1, 2, 3, 4, 5\}$, are independently examined for each data set. For the estimation of these hyperparameters, we have used grid search method with combinations of candidate values

$$\begin{aligned} \tau^2_\theta &= \{0.2, 0.1, 0.09, 0.08, 0.07, 0.06, \\ &\quad 0.05, 0.04, 0.03, 0.02, 0.01\}, \\ \tau^2_r &= \{0.1, 0.01, 0.005, 0.001, 0.0005, 0.0001\}. \end{aligned}$$

Leading 40 observations are excluded from the calculation of log-likelihood (14) to avoid unexpected effect of initial value. According to the value of log-likelihood, the optimal combination of hyperparameters has been determined. From the maximum log-likelihood obtained by the determined hyperparameters, the values of AIC are obtained for all m as shown in table 1. The AIC values of optimal number of peaks determined by minimum AIC method is underlined in the table.

For the optimal number of peaks, estimated median of peak frequencies have been plotted in figure 1 for 3-peaks-data and figure 2 for 4-peaks-data. In these figures, the true frequencies are denoted by solid lines together with the estimated peaks in each figures.

4.3 Comparison to TVCAR model

For a comparison of the performance of TVPP model, TVCAR model is applied to the same data sets. In TVCAR model, smoothness is assumed to the AR coefficients by taking a state vector

$$\mathbf{x}_t = \mathbf{a}_t. \qquad (17)$$

System noise vector $\mathbf{w}_t$ is normal distribution with diagonal covariance matrix of same variance τ^2 for all items.

For the order of TVCAR model, $p = 1, \cdots, 10$ are independently applied. For each AR order, $\tau^2 = \{10^{-q} | q = 1, \cdots, 9\}$ have been examined to search the optimal value of hyperparameter τ^2. Where, leading 40 observations are excluded from the calculation of log-likelihood to accomplish the same condition to TVPP model. Pre-filtering for the leading part of series has also done with initial condition that all coefficients are 0.

The hyperparameter τ^2 is estimated based on log-likelihood for each order, and AIC values obtained from them are shown in table 2. According to the minimum value of AIC, the optimal AR order has been determined for each data set. By looking the table, the correct order is not always selected for each data set. We have shown the minimum AIC by †, the true order by underline, in the table.

Peak frequency of TVCAR model is calculated from the estimated AR coefficient by using double-QR method, and they have been plotted in figures 3 and 4 for 3-peaks-data and 4-peaks-data respectively. In these figures, the true frequencies are denoted by solid lines.

Looking both AIC results by TVPP model and TVCAR model shown in tables 1 and 2, we can see that the TVPP model are superior to TVCAR model for all data sets. As the other criterion, Mean Squares Error(MSE) of peak frequency of power spectrum are calculated by

$$\mathrm{MSE} = \frac{1}{(N-40)m} \sum_{t=41}^{N} \sum_{k=1}^{m} \left[\hat{\theta}_{k,t} - \theta^*_{k,t} \right]^2 \qquad (18)$$

where, $\hat{\theta}_{k,t}$ is estimated value of peak frequency, and $\theta^*_{k,t}$ is the true value of peak frequency. The values of MSE are shown in table 3. By looking the table, TVPP model is also better than TVCAR model for all data sets.

Table 1: AIC of TVPP model

	data set	
m	3-peaks-data	4-peaks-data
1	892.24	1123.74
2	854.22	945.94
3	<u>829.56</u>	884.16
4	840.59	<u>840.89</u>
5	843.84	854.86

Table 2: AIC of TVCAR model

	data set	
p	3-peaks-data	4-peaks-data
1	1079.72	1125.53
2	887.30	1043.89
3	880.36	1032.91
4	856.90	943.61
5	864.71	<u>943.51</u>
6	<u>853.43</u>	948.32
7	858.26	949.67
8	863.68	†948.34
9	872.42	950.73
10	878.34	954.54

Table 3: Mean squares error of estimated peak[rad^2]

model	data set	
	3-peaks-data	4-peaks-data
TVPP	0.6162×10^{-2}	0.8006×10^{-2}
TVCAR	0.1656×10^{-1}	†0.3364×10^{-1}

5. Conclusion

We have proposed a method to estimate multiple peaks of power spectrum varying with time by Time-Varying Peak frequencies of Power spectrum (TVPP) model [5]. The model can be written in nonlinear state space representation in which the state vector consists of pairs of peak frequency and bandwidth of nonstationary power spectrum. The state can be estimated by nonlinear non-Gaussian filtering method. As the filtering method, we have employed an approximation by realization particles of the non-Gaussian distribution called Monte Carlo Filter (MCF) [9]. By applying MCF to TVPP model, a more number of peaks than that in conventional research [5] can be estimated. In a simulational experiment, the cases of 3 and 4 peaks have been examined. Comparison to Time-Varying Coefficient AutoRegressive(TVCAR) model [10] has been done. We have obtained the result that TVPP model is superior to TVCAR model with respect to AIC and mean squares error of peak frequency. For further work, comparison to the other conventional methods such as Wigner distribution [11] can be considered.

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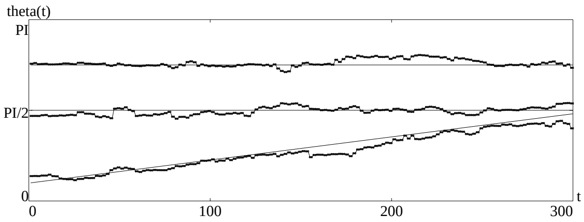

Figure 1: Estimated peak frequencies by TVPP model(median) for 3-peaks-data

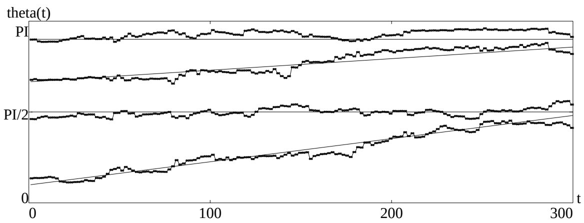

Figure 2: Estimated peak frequencies by TVPP model(median) for 4-peaks-data

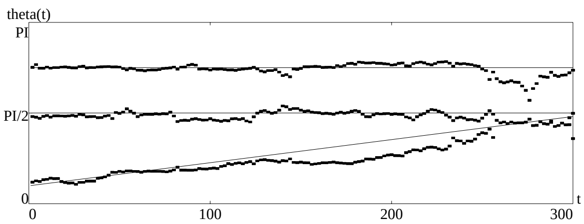

Figure 3: Estimated peak frequencies by TVCAR model for 3-peaks-data

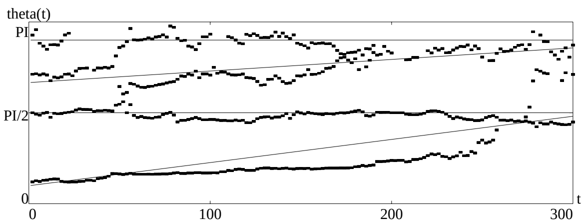

Figure 4: Estimated peak frequencies by TVCAR model for 4-peaks-data