Maneuvering target tracking by using particle filter method with model switching structure
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Abstract. Tracking problem of maneuvering target is treated with assumption that the maneuver is unknown and its acceleration has abrupt changes sometimes. To cope with unknown maneuver, Bayesian switching structure model, which includes a set of possible models and switches among them, is used. It can be formalized into general (nonlinear, non-Gaussian) state space model where system model describes the target dynamics and observation model represents a process to observe the target position. Heavy-tailed uni-modal distribution, e.g. Cauchy distribution, is used for the system noise to accomplish good performance of tracking both for constant period and abrupt changing time point of acceleration. Monte Carlo filter, which is a kind of particle filter that approximates state distribution by many particles in state space, is used for the state estimation of the model. A simulation study shows the efficiency of the proposed model by comparing with Gaussian case of Bayesian switching structure model.

Keywords. Bayesian modeling, target tracking, non-Gaussian distribution, multiple model, switching structure, particle filter.

1 Introduction

Problem of target tracking has been investigated actively after Kalman filter algorithm had been proposed, e.g.[Singer 1970]. Since the middle of 1980s, solution to this problem have been applied to, e.g., beam pointing control of a phased array radar, where benchmark problem is presented by [Blair and Watson 1994]. In this application, interacting multiple model that includes constant velocity model, constant thrust model and constant speed turn model is used with Kalman filter for state estimation [Blom and Bar-Shalom 1988].

Recently, a state estimation methods for nonlinear non-Gaussian state space model, which are called particle filter in general, are proposed: [Gordon et al. 1993], [Kitagawa 1996], and [Isard and Blake 1998]. These particle filters use many number of particles in state space to approximate non-Gaussian distribution of state estimate. Their ideas are considered as the special realization of sequential Monte Carlo method[Liu and Chen 1998]. For nonlinear or non-Gaussian model, particle filter can achieve more precise estimation of the state than the one of Kalman filter since Kalman filter only approximates

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the state distribution by Gaussian(uni-modal) while the actual one might be multi-modal.

The particle filters allows us to use nonlinear structure for the target tracking problem. Bayesian switching structure model that includes a set of possible models is applied to the problem and good performance is reported in [McGinnity and Irwin 2001]. Bayesian switching is also related to self organizing model[Kitagawa 1998] that automatically tune the hyper-parameters of the model by augmenting the state vector with hyper-parameters. This idea is generalized to switching the model structure by adding indicator vector of the model to the state vector [Higuchi 2001].

While simultaneous consideration of the multiple models is effective for dealing with the target tracking problem, non-Gaussian distribution for describing the system noise is also worthy of being considered. A use of heavy-tailed uni-modal distribution to follow abrupt changes of target's acceleration is proposed with particle filter[Ikoma et al. 2001]. A representative one of such distribution is Cauchy distribution. By assuming Cauchy distribution to the system noise, which corresponds to increments of acceleration, good performance of tracking is accomplished both for constant period and abrupt changing time point of acceleration.

In this paper, we propose the use both of Bayesian switching structure and heavy-tailed uni-modal distribution simultaneously in a tracking problem. A simulation study shows the efficiency of the proposed model by comparing with Gaussian system noise case of Bayesian switching structure model.

2 Model

Basic model for target tracking is firstly introduced, where, acceleration is assumed to be constant in continuous model, i.e., first derivative of the constant assumed element is according to a uni-modal distribution of 0 mode. The basic model is extended to Bayesian switching structure model that includes a set of possible models(candidate models), e.g., constant velocity model, constant acceleration model, and so on, with state vector that consists of position, velocity, acceleration, and jerk (and higher derivatives if needed) of the target.

To cope with the unknown maneuver, the state vector is extended to include indicator variable to select one model among the possible models. Markov switching is used to allow the indicator variable to evolve in the system model. To follow abrupt change of acceleration, we assume heavy-tailed uni-modal distribution for system noise. It will achieve good performance both for constant acceleration period and abrupt changing time point due to the heavy-tail property. The model is formalized as a nonlinear non-Gaussian state space model with system model described above and observation model that represents position observation process of the target.

2.1 Basic model

Position of the target in one-dimensional space is treated here, and is represented by $r(t)$ where t stands for continuous time index. Let acceleration of the target, $a(t)$, be a maneuver and it is assumed to be unknown. System model, which describes dynamics of the target, can be written in stochastic differential equation

$$\begin{bmatrix} \dot{r}(t) \\ \dot{s}(t) \\ \dot{a}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r(t) \\ s(t) \\ a(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_a(t) \qquad (1)$$

where $s(t)$ is velocity of the target, and $v_a(t)$ is white Gaussian noise with 0 mean and variance ${\tau_a}^2$.

By discretizing the continuous system model eq.(1) with sampling time T (i.e. sampling points becomes $t = T_0 + kT$ with discrete time index k), with 0-th order hold assumption to the system noise such that $v_k^{(a)} = v_a(kT)$, and by denoting $r_k = r(kT)$, $s_k = s(kT)$, $a_k = a(kT)$, we have a discrete time system model

$$\begin{bmatrix} r_k \\ s_k \\ a_k \end{bmatrix} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{k-1} \\ s_{k-1} \\ a_{k-1} \end{bmatrix} + \begin{bmatrix} T^3/3! \\ T^2/2 \\ T \end{bmatrix} v_k^{(a)}. \tag{2}$$

Note that acceleration of eq.(2) is modeled by random walk, and effects of the random walk model's terms($a(t)$ and $v_a(t)$) during the sampling time T appear in transition matrix and the vector apearing in the second term of right hand side in eq.(2).

From the state of the system, observation y_k is target position corrupted with observation noise w_k,

$$y_k = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} r_k \\ s_k \\ a_k \end{bmatrix} + w_k \tag{3}$$

where w_k is assumed to be Gaussian white with 0 mean and variance σ^2.

2.2 Switching

Due to the assumption that maneuver of the target is unknown, acceleration of the target, which is element of random walk in system model eq.(2), may be 0 for some time, may have some certain non-zero value for the another time, and may have changes between these values. This causes model mismatching to apply eq.(2) while acceleration constant period. To cope with this, we prepare candidate models of different element of random walk, i.e., position, velocity, acceleration, jerk(difference of acceleration) and so on, and switch system model among them.

Candidate models are as follows. Firstly, position constant(random walk) model is

$$r_k = r_{k-1} + T v_k^{(r)} \tag{4}$$

where $v_k^{(r)} = v_r(kT)$ with white Gaussian system noise $v_r(t)$ with 0 mean and variance ${\tau_r}^2$. Next, velocity constant(random walk) model is

$$\begin{bmatrix} r_k \\ s_k \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_{k-1} \\ s_{k-1} \end{bmatrix} + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} v_k^{(s)} \tag{5}$$

where $v_k^{(s)} = v_s(kT)$ with white Gaussian system noise $v_s(t)$ with 0 mean and variance ${\tau_s}^2$. Finally, jerk constant(random walk) model is

$$\begin{bmatrix} r_k \\ s_k \\ a_k \\ c_k \end{bmatrix} = \begin{bmatrix} 1 & T & T^2/2 & T^3/3! \\ 0 & 1 & T & T^2/2 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{k-1} \\ s_{k-1} \\ a_{k-1} \\ c_{k-1} \end{bmatrix} + \begin{bmatrix} T^4/4! \\ T^3/3! \\ T^2/2 \\ T \end{bmatrix} v_k^{(c)} \tag{6}$$

where, $c_k = c(kT)$ stands for jerk, and $v_k^{(c)} = v_c(kT)$ with white Gaussian system noise $v_c(t)$ with 0 mean and variance ${\tau_c}^2$.

Among these candidate models(eq. (2), (4), (5), and (6)), one is selected and used as a system model. To denoted the selection, an indicator that specifies the selected model is introduced into the state of the model. The indicator is denoted by i_k, and its value is equal to the highest order of the element, i.e., 1 means constant position, 2 velocity, 3 acceleration, and 4 jerk. It is switched according to Markov process with transition matrix (which consists of transition probability), for example(in case of four candidate models),

$$\Pr\{i_k = i | i_{k-1} = j\} = \begin{bmatrix} 0.950 & 0.025 & 0.000 & 0.000 \\ 0.050 & 0.950 & 0.025 & 0.000 \\ 0.000 & 0.025 & 0.950 & 0.050 \\ 0.000 & 0.000 & 0.025 & 0.950 \end{bmatrix} \tag{7}$$

where column correspond to the indicator value before transit, and row corresponds to the after.

2.3 Heavy-tailed system noise

Since the property of unknown maneuver, acceleration of the target may have abrupt change. It is represented by system noise term $v_k^{(a)}$ in acceleration constant model eq.(2). With Gaussian system noise, its variance must be increased to follow this abrupt change, however, on the other hand, stability for constant acceleration period will be lost. To satisfy both properties of following and stability simultaneously, uni-modal heavy-tailed distribution is employed for system noise. Where, uni-mode represents the small fluctuation with high probability for stable period and heavy-tail bears abrupt change with low probability. Cauchy distribution is typical for such distribution, and we use it in simulation study.

3 Simulation study

One-dimensional trajectory shown in Fig.1(a) is used, which has small observation noise with $N(0, \sigma^2)$, $\sigma^2 = 10^{-4}$. Acceleration of the trajectory is shown in Fig.1(b). In all figure, horizontal axis shows discrete time index k.

By applying Bayesian switching structure model for Gaussian case(ordinary) and Cauchy(our proposal), we have obtained the estimate of position, velocity, acceleration, and jerk. As for the condition of Monte Carlo filter, number of particles is set to 50,000. system noise variances ${\tau_r}^2$, ${\tau_s}^2$, ${\tau_a}^2$, and ${\tau_c}^2$ are determined by grid search for maximizing the likelihood of the data.

Acceleration result(median) for both model are shown in Fig.2 by solid line, with actual one by dashed line. It can be seen that Cauchy estimates keeps stable at constant acceleration period without loss of following property of abrupt changing points.

Evolution of model indicator variable, which is involved in state vector together, for both model are shown in Fig.3. Indicator value 1(or 1 and 2) are majority for beginning part(i.e., constant position), 3 is major in the middle of the series, 2 is major at the ending part(constant velocity). By looking the result, the most appropriate model is majority almost all the period of the series.

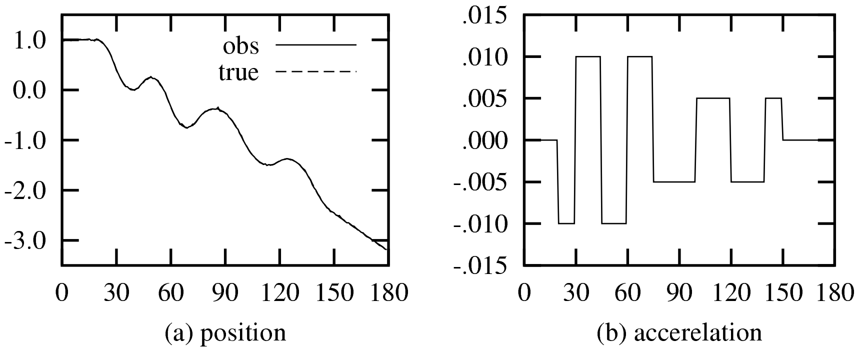

Fig. 1. Trajectory of target, (a)position(observation and true) and (b)acceleration.

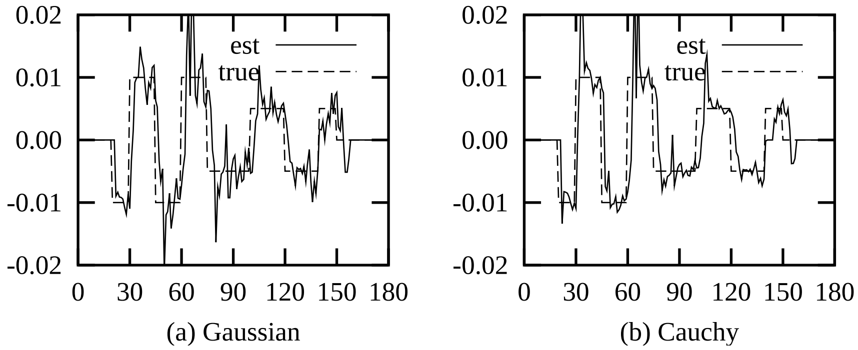

Fig. 2. Acceleration result: median (a)Gaussian and (b)Cauchy.

4 Conclusion

By exploiting an advantage of particle filter that allows us to treat nonlinear and non-Gaussian time series model tractably, we propose the use both of Bayesian switching structure(nonliner) and heavy-tailed uni-modal distribution(non-Gaussian) simultaneously in a target tracking problem. A simulation study that treats simple one-dimensional space tracking problem shows the efficiency of the proposed model by comparing with Gaussian system noise case of Bayesian switching structure model.

The model can easily be extended to multi-dimensional position with nonlinear observation equation by radar(polar coordinate). It will be an interesting future work.

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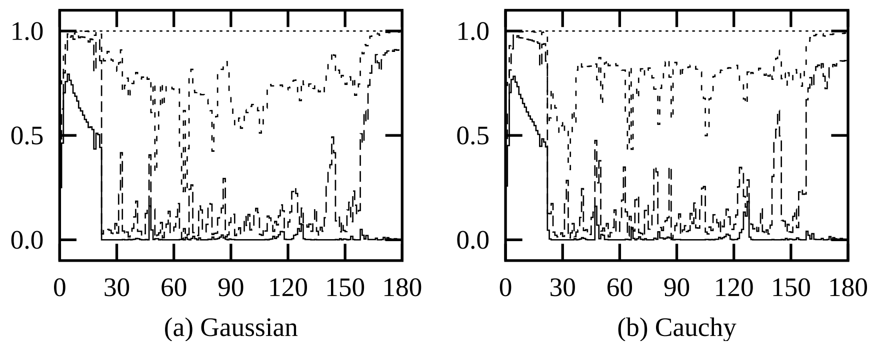

Fig. 3. Switching result: accumulated ratio of possible models (a)Gaussian and (b)Cauchy.

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