EXTENDED OBJECT TRACKING WITH UNKNOWN ASSOCIATION, MISSING OBSERVATIONS, AND CLUTTER USING PARTICLE FILTERS

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ABSTRACT

A new method for target tracking of multiple points on an object by using particle filter with its novel importance function is proposed. The assumptions are such that the number of points is fixed and known, and the association between points of object and observed points are unknown. There exists missing and clutter in observation process where which observation corresponds to them are also unknown. The main difficulty of this problem is the formidable number of combinations in the association. The novel importance function using an idea of soft gating makes the problem tractable in a proper framework of particle filter. Simulation experiment illustrates the performance of the method.

1. INTRODUCTION

Target tracking is one of the most classical and important applications of filtering techniques. Many models have been proposed to deal with various tracking situations since the Kalman filter was proposed. Recently, computer intensive methods for filtering, called "particle filters" [3] [6] or sequential Monte Carlo (SMC) [1] [7], have been developed and used for the tracking problem, see, e.g., [8].

In the current researches, multiple target tracking, e.g. [4], is a challenging problem where associations between measurements and the state are unknown. The problem setting is that several points on an object can be observed due to high resolution of sensor but there is no information about that which observation corresponds to which point on the object. We call the object with multiple points as "extended object". The main difficulty of the problem is the formidable number of combinations in the association. For example, if the number of targets is fixed as 10, the number of permutations $10! = 3,628,800$ is already large, furthermore, the number of combination will increase when there are missing data and clutter in the observation process.

To deal with this huge number of combinations, a traditional idea called gating is to select the meaningful associations based on the information in observed data. An extension of this idea to particle filters was proposed in [4], where hypotheses of the association are selected by gating while the target's state is estimated by particle filters. On the other hand, [5] proposed a full implementation of particle filters in a general framework of the jump Markov model [2], in which full estimation is carried out for both the target's state and the association.

In this paper, we propose a new method to deal with tracking of an extended object under the condition of unknown associations, missing observations, and clutter. The method uses a full implementation of particle filters where all points on the object share the same velocity of the object. The novelty of the method is on a new sub-optimal importance function for the association using the ideal of soft gating. This formulation allows us to use the gating idea in a proper SMC framework, see [5] for related topics, and overcomes the association problem without having to enumerate all states, or introducing further approximation. Simulation experiments will illustrate the performance of the proposed method.

2. MODEL

Consider that N_T points are on the object in a d-dimensional space. In real situation, , for example, this is due to high resolution of the sensor that can resolve the points on an object. Some of the points on the object, i.e. not necessarily all, are observed for each time. The observations may contain clutter points, which are wrong detection of points at non-existing positions of the target due to bad sensing conditions. Assume that we know neither which observed point corresponds to which point of the object, nor which observed point is the clutter. Here we assume that N_T is fixed and known.

Let k be an integer denoting the discrete time index of observations, and the number of observed points at time k, N_k, is given together with the observation. At time k, let $N_{D,k} \leq \min(N_T, N_k)$ be the number of detected points of the target, and let $N_{C,k}$ be the number of clutter points, thus $N_k = N_{D,k} + N_{C,k}$ holds.

To summarize, both $N_{D,k}$ and $N_{C,k}$ are assumed to be

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unknown, as well as association between observed points and target points are unknown.

2.1. System model

Let $\mathbf{x}_{j,k}$ be a d-dimensional vector that denotes j-th point of the target at time k where the origin of the vector is at the center of gravity of the target, so $\sum_{j=1}^{N_T} \mathbf{x}_{j,k} = 0$ holds.

Let $\mathbf{c}_k$ be a d-dimensional vector that represents the center of gravity of the target points at time k. Suppose that $\mathbf{c}_k$ moves according to the following dynamics:

$$\begin{bmatrix} \mathbf{c}_k \\ \dot{\mathbf{c}}_k \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{I} \\ \mathbf{O} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{c}_{k-1} \\ \dot{\mathbf{c}}_{k-1} \end{bmatrix} + \begin{bmatrix} \mathbf{O} \\ \mathbf{I} \end{bmatrix} \mathbf{v}_k^c \tag{1}$$

where $\mathbf{v}_k^c \sim N(\mathbf{0}, \tau_c^2 \mathbf{I})$ is the system noise, representing uncertainty about the maneuver of the target by a Gaussian distribution.

When we assume a rigid object as the target, the dynamics of each point of the target are supposed to be constant with respect to time such that $\mathbf{x}_{j,k} = \mathbf{x}_{j,k-1}$ for $j = 1, 2, \cdots, N_T$. Otherwise, the dynamics involve system noise terms $\mathbf{v}_{j,k}^x \sim N(0, \tau_x^2 \mathbf{I})$ as $\mathbf{x}_{j,k} = \mathbf{x}_{j,k-1} + \mathbf{v}_{j,k}^x$ for $j = 1, 2, \cdots, N_T$.

By formulating a state vector as

$$\mathbf{x}_k = \left[\mathbf{x}_{2,k}^T, \cdots, \mathbf{x}_{N_T,k}^T, \mathbf{c}_k^T, \dot{\mathbf{c}}_k^T\right]^T, \tag{2}$$

where $\mathbf{x}^T$ denotes transpose of $\mathbf{x}$, we have a system model of the problem in the form of a linear Gaussian difference equation as

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{G}\mathbf{v}_k \tag{3}$$

with proper definition of matrices $\mathbf{F}$ and $\mathbf{G}$ as described above.

2.2. Unknown Association

Let $\tilde{\mathbf{y}}_{j,k}$ be the true position of j-th point of the target; then we have

$$\tilde{\mathbf{y}}_{j,k} = \mathbf{x}_{j,k} + \mathbf{c}_k, \tag{4}$$

for $j = 1, 2, \cdots, N_T$. On the other hand, the d-dimensional observation at time k is denoted by $\mathbf{y}_{j,k}$ for $j = 1, 2, \cdots, N_k$. According to the assumption of unknown associations, we know neither which observation is clutter nor which observation corresponds to the points on the object.

To deal with the unknown association between the target points and the observed points, we introduce an association vector that consists of N_k-tuples of integers such that

$$\mathbf{I}_k = [I_{1,k}, I_{2,k}, \cdots, I_{N_k,k}]^T, \tag{5}$$

where $I_{j,k} \in \{0, 1, \cdots, N_T\}$ for $j = 1, 2, \cdots, N_k$. Here, $I_{j,k} = i > 0$ means j-th observation comes from i-th point of the target, and $I_{j,k} = 0$ means j-th observation is a clutter. In the next subsection, the association vector specifies the association between observed points and target points in the observation equation.

2.3. Observation model

At time k, let $\mathbf{y}_k$ be an observation vector that consists of $\mathbf{y}_{j,k}$ for $j = 1, 2, \cdots, N_k$, and let $\mathbf{w}_k$ be a vector of observation noise that consists of $\mathbf{w}_{j,k}$ for $j = 1, 2, \cdots, N_k$. Each $\mathbf{w}_{j,k}$ is assumed to be a Gaussian random vector with zero mean and identity covariance matrix, both mutually independent and independent of the other variables. Now, we can write the observation process by

$$\mathbf{y}_{j,k} = \sum_{i=0}^{N_T} \delta_{i,I_{j,k}} \tilde{\mathbf{y}}_{i,k} + \sigma^2(I_{j,k}) \mathbf{w}_{j,k}, \tag{6}$$

where the symbol $\delta_{i,j}$ is the Kronecker's delta, $\tilde{\mathbf{y}}_{0,k} \equiv \bar{\mathbf{y}}$ is the central position of the surveillance range, and

$$\sigma^2(I_{j,k}) = \begin{cases} \sigma^2 & , \quad I_{j,k} \neq 0, \\ \alpha\sigma^2 & , \quad I_{j,k} = 0, \quad \alpha \gg 1 \end{cases} \tag{7}$$

is employed to deal with the clutter.

Now, observation model can be denoted in a general form by

$$\mathbf{y}_k = \mathbf{H}(\mathbf{I}_k)\mathbf{x}_k + \mathbf{D}(\mathbf{I}_k)\mathbf{w}_k \tag{8}$$

where $\mathbf{H}(\mathbf{I}_k)$ and $\mathbf{D}(\mathbf{I}_k)$ are matrices depending on the association vector $\mathbf{I}_k$ of eq.(5). These matrices are properly defined as described above.

Then, we can form a jump Markov linear system by pairing the observation model eq.(8) and the system model eq.(3). In the context of jump Markov linear systems, it is often assumed that $\mathbf{I}_k$ is a Markov process. However in our problem, it is independent, i.e., $P(\mathbf{I}_k|\mathbf{I}_{k-1}, \mathbf{I}_{k-2}, \cdots, \mathbf{I}_0) = P(\mathbf{I}_k)$ holds.

2.4. Assumptions for the observation process

We need further definitions and assumptions for the observation process on detection probability, probability of clutter, and permutation of observed points, as follows.

Let p_D be a probability of detection of each target, $\tilde{\mathbf{y}}_{j,k}$ for $j = 1, 2, \cdots, N_T$. The detection process is assumed to be independent for each target and independent of the target's state vector. Then, the number of detected points, $N_{D,k}$, is drawn according to a binominal distribution, i.e., $N_{D,k} \sim B(N_T, p_D)$.

The number of clutter points, $N_{C,k}$, is assumed to be drawn according to a Poisson distribution, i.e., $N_{C,k} \sim$ Poisson (μV), independently of time, k, where V is the volume of surveillance and μ is a spatial density of clutter.

The permutation of detected points and clutter points are assumed to be uniformly distributed, i.e. there are $N_k!$ combinations for it; each with the same probability $1/N_k!$.

3. STATE ESTIMATION BY PARTICLE FILTERS

The aim of filtering is to obtain the conditional distribution of the augmented state vector $\mathbf{z}_k = \left[\mathbf{x}_k^T, \mathbf{I}_k^T\right]^T$, given observations up to k, $\mathbf{y}_{1:k} \equiv (\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_k)$, that is, $p(\mathbf{z}_k|\mathbf{y}_{1:k})$. Note that it is obtained from $p(\mathbf{z}_{0:k}|\mathbf{y}_{1:k})$ with its marginal.

3.1. Particle filters

Particle filters use many (say M) particles $\{\mathbf{z}_{0:k}^{(l)}\}_{l=1}^M$ to approximate conditional distribution $p(\mathbf{z}_{0:k}|\mathbf{y}_{1:k})$, where particles $\{\mathbf{z}_{0:k}^{(l)}\}_{l=1}^M$ are considered as that drawn from the conditional distribution.

Particles are updated, when a new observation $\mathbf{y}_k$ becomes available, as follows [1], [7]. Assume that particles at time $k-1$, $\{\mathbf{z}_{0:k-1}^{(l)}\}_{l=1}^M$, are given. First, for $l = 1, 2, \cdots, M$, draw new particles $\tilde{\mathbf{z}}_k^{(l)}$ from a distribution, $\pi(\tilde{\mathbf{z}}_k|\mathbf{z}_{0:k-1}^{(l)}, \mathbf{y}_{1:k})$, which is called "importance function" or "proposal" distribution. And we let $\tilde{\mathbf{z}}_{0:k}^{(l)} = (\mathbf{z}_{0:k-1}^{(l)}, \tilde{\mathbf{z}}_k^{(l)})$. Then, calculate a weight for each particle by

$$\omega_k^{(l)} \propto p(\mathbf{y}_k|\tilde{\mathbf{z}}_{0:k}^{(l)}) p(\tilde{\mathbf{z}}_k^{(l)}|\mathbf{z}_{0:k-1}^{(l)}) / \pi(\tilde{\mathbf{z}}_k^{(l)}|\mathbf{z}_{0:k-1}^{(l)}, \mathbf{y}_{1:k}). \quad (9)$$

Finally, proceed by resampling from $\{\tilde{\mathbf{z}}_{0:k}^{(l)}\}_{l=1}^M$ with a probability proportional to the value of weight $\omega_k^{(l)}$, specifically, for $i = 1, 2, \cdots, M$, let $\mathbf{z}_{0:k}^{(i)} := \tilde{\mathbf{z}}_{0:k}^{(l)}$ with probability proportional to $\omega_k^{(l)}$. Then, we obtain particles, $\{\mathbf{z}_{0:k}^{(i)}\}_{i=1}^M$, which are approximately the particles drawn from $p(\mathbf{z}_{0:k}|\mathbf{y}_{1:k})$.

Actual computation proceeds only with $\{\mathbf{z}_k^{(i)}\}_{i=1}^M$, it is for marginal $p(\mathbf{z}_k|\mathbf{y}_{1:k})$, by discarding $\{\mathbf{z}_{0:k-1}^{(i)}\}_{i=1}^M$ in the above.

3.2. Rao-Blackwellization

Since the model is linear if $\mathbf{I}_{0:k}$ is given, we can use a variance reduction method called Rao-Blackwellization (RB) for the filtering [1]. Here, variance of the weight in eq.(9) with respect to the draw of $\tilde{\mathbf{z}}_k^{(l)}$ is reduced by RB method. Hence variance of the estimates derived from the conditional distribution $p(\mathbf{z}_{0:k}|\mathbf{y}_{1:k})$ will also be reduced. For more theoretical detail, see [1].

In RB method, the target distribution is decomposed as $p(\mathbf{z}_{0:k}|\mathbf{y}_{1:k}) = p(\mathbf{I}_{0:k}|\mathbf{y}_{1:k}) p(\mathbf{x}_{0:k}|\mathbf{I}_{0:k}, \mathbf{y}_{1:k})$, where the second distribution is Gaussian, so, for given $\mathbf{I}_{0:k}$, analytic solution can be obtained by Kalman filter based algorithm.

Actual computation proceeds as follows. $p(\mathbf{I}_k|\mathbf{y}_{1:k})$ is estimated by particle filters using particles $\{\mathbf{I}_k^{(l)}\}_{l=1}^M$ while $p(\mathbf{x}_k|\mathbf{I}_{0:k}^{(l)}, \mathbf{y}_{1:k})$ is obtained by a Kalman filter with its mean vector $\bar{\mathbf{x}}_{k|k}^{(l)}$ and covariance matrix $\mathbf{V}_{k|k}^{(l)}$. Let the particles at time $k-1$ denoted by $\{\mathbf{I}_{k-1}^{(l)}\}_{l=1}^M$ be given, and let the mean vector $\bar{\mathbf{x}}_{k-1|k-1}^{(l)}$ and covariance matrix $\mathbf{V}_{k-1|k-1}^{(l)}$ for $l = 1, 2, \cdots, M$ also be given. Then, update procedure for a given observation $\mathbf{y}_k$ proceeds as follows; First $\tilde{\mathbf{I}}_k^{(l)}$ for $l = 1, 2, \cdots, M$ are drawn from a proposal $\pi(\tilde{\mathbf{I}}_k|\mathbf{I}_{0:k-1}^{(l)}, \mathbf{y}_{1:k})$. Next, the mean vector and the covariance matrix are are updated to k by Kalman filter procedure, then we obtain the mean vector $\tilde{\bar{\mathbf{x}}}_{k|k}^{(l)}$ and covariance matrix $\tilde{\mathbf{V}}_{k|k}^{(l)}$ of the Gaussian distribution $p(\mathbf{x}_k|\tilde{\mathbf{I}}_k^{(l)}, \mathbf{I}_{0:k-1}^{(l)}, \mathbf{y}_{1:k})$. Where, we can obtain the likelihood of $\mathbf{y}_k$, $p(\mathbf{y}_k|\tilde{\mathbf{I}}_k^{(l)}, \mathbf{I}_{0:k-1}^{(l)}, \mathbf{y}_{1:k-1})$, through the Kalman filter procedure. Now we can calculate the weights for new particles

$$\omega_k^{(l)} \propto \frac{p(\mathbf{y}_k|\tilde{\mathbf{I}}_k^{(l)}, \mathbf{I}_{0:k-1}^{(l)}, \mathbf{y}_{1:k-1}) P(\tilde{\mathbf{I}}_k^{(l)}|\mathbf{I}_{0:k-1}^{(l)})}{\pi(\tilde{\mathbf{I}}_k^{(l)}|\mathbf{I}_{0:k-1}^{(l)}, \mathbf{y}_{1:k})}. \quad (10)$$

By resampling from a set with elements be tuples of particle, mean vector, and covariance matrix, $\left\{ \left(\tilde{\mathbf{I}}_k^{(l)}, \tilde{\bar{\mathbf{x}}}_{k|k}^{(l)}, \tilde{\mathbf{V}}_{k|k}^{(l)}\right) \right\}_{l=1}^M$, we obtain particles $\{\mathbf{I}_k^{(i)}\}_{i=1}^M$ together with $\bar{\mathbf{x}}_{k|k}^{(i)}$ and $\mathbf{V}_{k|k}^{(i)}$ for $i = 1, 2, \cdots, M$.

3.3. Importance function

Design of the importance function is a key for particle filters to have efficient estimation results. Here we suggest a novel importance function of the following form

$$\begin{aligned} \pi(\mathbf{I}_k|\mathbf{x}_{k-1}, \mathbf{y}_k) &= \pi(I_{1,k}|\mathbf{x}_{k-1}, \mathbf{y}_{1,k}) \\ &\times \pi(I_{2,k}|\mathbf{x}_{k-1}, \mathbf{y}_{2,k}, I_{1,k}) \\ &\vdots \\ &\times \pi(I_{N_k,k}|\mathbf{x}_{k-1}, \mathbf{y}_{N_k,k}, I_{1,k}, I_{2,k}, \cdots, I_{N_k-1,k}), \end{aligned} \quad (11)$$

where $\pi(I_{1,k}|\mathbf{x}_{k-1}, \mathbf{y}_{1,k})$ is proportional to

$$\frac{1}{\sigma(I_{1,k})} \exp\left\{ -\frac{1}{2\sigma^2(I_{1,k})} \left| \mathbf{y}_{1,k} - \sum_{i=0}^{N_T} \delta_{i,I_{1,k}} \tilde{\mathbf{y}}_{i,k} \right|^2 \right\}, \quad (12)$$

and $\pi(I_{2,k}|\mathbf{x}_{k-1}, \mathbf{y}_{2,k}, I_{1,k})$, $\pi(I_{3,k}|\mathbf{x}_{k-1}, \mathbf{y}_{3,k}, I_{1,k}, I_{2,k})$, $\cdots$ are similar form except there is no probability for $I_{j,k}$ to take the same value of $I_{i,k}$, for $i < j$.

The proposed method uses a soft gating function in a proper SMC framework, thus it overcomes the association problem without enumerating all states, or introduce further approximation. See [5] for an alternative use of gating ideas with SMC.

4. SIMULATION EXPERIMENT

Two simulation experiments have been carried out to illustrate how the proposed method works. One is 1-dimensional tracking for $N_T = 10$ target points of $k = 1, \cdots, 30$ with $P_D = 0.75$, $V = 15$, $\mu = 0.2$ and $\sigma^2 = 10^{-2}$. The other is tracking in 3-dimension for $N_T = 5$ points of $k = 1, \cdots, 50$ with $P_D = 0.5$, $V = 3.375 \times 10^7$, $\mu = 0.05 \times$

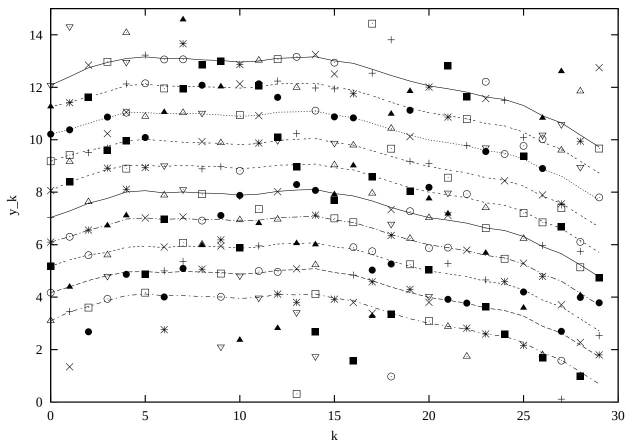

Fig. 1. Estimation results of 1-dimension for 10 points.

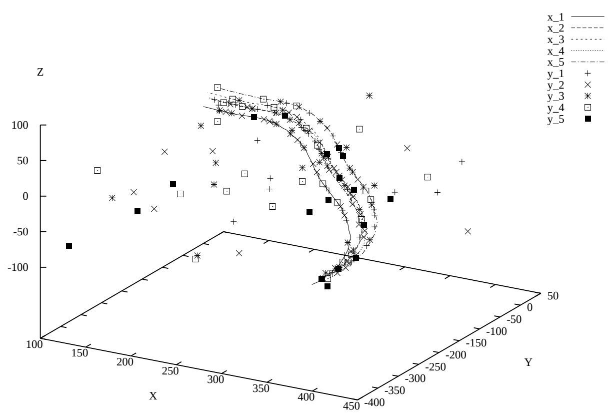

Fig. 2. Estimation results of 3-dimension for 5 points.

10^{-7} and $\sigma^2 = 0.1$. Rigid object moving without rotation is assumed here for both cases.

Conditions of state estimation are as follows. For 1-dimensional case, $M = 10,000$ particles are used with system noise variance $\tau_c^2 = 10^{-2}$, and clutter factor $\alpha = 10^8$. For 3-dimensional case, $M = 1,000$ particles are used with $\tau_c^2 = 1$ and $\alpha = 10^6$.

Estimation results are shown in **Fig.**1 and **Fig.** 2. In these figures, plotted symbols represent the observation data, and lines denote the estimation result which is the mean of the estimated distribution. Estimation results of associations for the beginning part of the 3-dimensional data are shown in **Table** 1. In results of **Fig.**1 and **Fig.**2, true data are almost identical to the estimated states, although they are not plotted in these figures.

Table 1. Estimation results of association.

k	estimated association (true association)						
1	2 (2)	5 (4)	3 (3)	0 (5)	1 (1)		
2	1 (1)	4 (4)	2 (2)	3 (3)	5 (5)		
3	4 (4)	2 (2)					
4	5 (5)	4 (4)	3 (3)	2 (2)			
5	1 (1)	0 (0)	5 (5)	0 (0)	0 (0)	0 (0)	3 (3)
6	3 (3)	0 (0)	0 (0)				
7	5 (5)	4 (4)	1 (1)				
8	0 (0)	4 (4)	0 (0)	0 (0)	0 (0)		
9	5 (5)	2 (2)	0 (0)	3 (3)	4 (4)		
10	5 (5)	3 (3)	0 (0)	0 (0)	0 (0)		
11	1 (1)	0 (0)	5 (5)				
12	0 (0)	1 (1)	4 (4)	5 (5)	0 (0)		

5. CONCLUSION

A new method for tracking of multiple point targets of extended object by particle filter with elaborated importance function has been proposed. Simulations illustrate performance of the method for a rigid object in parallel motion case, with simultaneous estimation of the points trajectories and the associations in a particle filter framework.

For future researches, extensions of the proposed model to involve rotation of object and to bearings only observations, using an extended Kalman filter for both cases, are interesting. Further extensions to deal with multiple objects independently moving are more challenging and also interesting for future research.

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