Tracking of maneuvering target by using switching structure and heavy-tailed distribution with particle filter method

Norikazu Ikoma[1], Tomoyuki Higuchi[2], and Hiroshi Maeda[1]

[1] Faculty of Engineering, Kyushu Institute of Technology, Fukuoka 804-8550, Japan
[2] The Institute of Statistical Mathematics, Tokyo 106-8569, Japan

Abstract Tracking problem of maneuvering target with assumption that the maneuver is unknown and its acceleration has some abrupt changes is treated by formulating general (nonlinear, non-Gaussian) state space model with system model to describe the target dynamics and observation model to represent a measurement process of the target position. Bayesian switching structure model, which includes a set of possible models and switches among them, is used to cope with the unknown maneuver. Heavy-tailed uni-modal distribution, e.g. Cauchy distribution, is also used for the system noise to accomplish good performance of tracking both for constant period and abrupt changing time point of acceleration. Monte Carlo filter, which is a kind of particle filter that approximates state distribution by many particles in state space, is used for the state estimation of the model. A simple simulation study shows an improvement of performance by the proposed model comparing with Gaussian case of Bayesian switching structure model.
Keywords: Target tracking, switching structure, particle filter, non-Gaussian distribution.

1 Introduction

Target tracking problem can be applied to various control problems, e.g., beam pointing control of a phased array radar, where benchmark problem is presented by [4]. It has been investigated actively, especially after Kalman filter algorithm proposed, e.g.[15]. It uses a state space model, where dynamics of the target is written by system model and measurement process of the target's position is represented by observation model. However in case of manned maneuvering target, there are some changes of the target's acceleration at the unpredictable timing. Due to the change of acceleration, mismatching of the model to the target will occur, i.e., use of acceleration variable model to the constant acceleration situation makes unstable estimation, on the other hand, applying constant acceleration model when the actual acceleration is changed gives bad performance.

To overcome this, a use of Bayesian switching structure model that includes a set of possible models is effective. A realization of this is interacting multiple model that includes constant velocity model, constant thrust model and constant speed turn model with Kalman filter for state estimation [5]. Bayesian switching is also related to self organizing model [11] that automatically tune the hyper-parameters of the model by augmenting the state vector with hyper-parameters. This idea is generalized to switching the model structure by adding indicator vector of the model to the state vector [7].

On the other hand, according to the recent investigations of state estimation, several methods for nonlinear non-Gaussian state space model are proposed [6], [12], [9]. They are called "particle filter" in general, because of their approximation of non-Gaussian distribution of the state by many number of particles in state space. This idea is considered as the special realization of sequential Monte Carlo method [13]. Particle filter can make more precise estimation of the state than the Kalman filter for nonlinear or non-Gaussian model, since Kalman filter only approximates the state distribution by Gaussian(uni-modal) while the actual one might be multi-modal. By using particle filter, we can use nonlinear structure model for the target tracking problem. There is a report that shows an improvement of target tracking performance of Bayesian switching structure model with particle filter [14].

Property of non-Gaussian distribution is also useful to improve the performance of target tracking. By assuming heavy-tailed uni-modal distribution such as Cauchy distribution to the system noise, abrupt changes of the target's acceleration can be tracked without losing stability of constant acceleration period [8]. The reason of this is that uni-mode and heavy-tail of the distribution respectively represent usual small fluctuation and abrupt change of acceleration in the simultaneous manner. By using particle filter, useful property of non-Gaussian distribution like this situation can be introduced to the model building.

In this paper, we propose to use both of Bayesian switching structure and heavy-tailed uni-modal distribution in the target tracking problem. A simple simulation study shows the performance improvement of the proposed model by comparing with Gaussian system noise case of Bayesian switching structure model.

2 Model

One dimensional target tracking problem is modeled here. Random walk with respect to acceleration of the target is firstly defined as a basic model. Since random walk means almost constant at small time interval rather than walking randomly in the context of this research, we call this basic model "acceleration constant model".

After that, the basic model is extended to Bayesian switching structure model that includes a set of possible models(candidate models) such as constant velocity model, constant acceleration model, and so on. State vector is also extended up to highest order of derivative required to the candidate models, i.e., position, velocity, acceleration, and jerk (and higher derivatives if needed) of the target.

To switch among the candidate models, the state vector is augmented to include indicator variable to select one model among the candidates. Markov switching is used to allow the indicator variable to evolve in the system model. We assume heavy-tailed uni-modal distribution for system noise to follow abrupt change of acceleration.

2.1 Basic model

Let $r(t)$ be position, $s(t)$ be velocity, and $a(t)$ be acceleration of the target at time t. Where t stands for continuous time index. Acceleration of the target is a maneuver and it is assumed to be unknown. Dynamics of the target is described by system model. It can be written in stochastic differential equation

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}v_a(t) \tag{1}$$

where $\mathbf{x}(t)$ is state vector

$$\mathbf{x}(t) = [\, r(t) \;\; s(t) \;\; a(t) \,]^t, \tag{2}$$

$\mathbf{A}$ is state transition matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \tag{3}$$

vector $\mathbf{b}$ is defined as

$$\mathbf{b} = [\, 0 \quad 0 \quad 1 \,]^t, \tag{4}$$

and $v_a(t)$ is white Gaussian noise with 0 mean and variance ${\tau_a}^2$.

By applying time discretization to eq.(1) with sampling time T (i.e. sampling point is $t = T_0 + kT$ with discrete time index k), we have a discrete time system model

$$\mathbf{x}_k = \mathbf{F}_a\mathbf{x}_{k-1} + \mathbf{g}_a v_k^{(a)}. \tag{5}$$

Here we assume 0-th order hold to the system noise such that $v_k^{(a)} = v_a(kT)$, and use notations $r_k = r(kT)$, $s_k = s(kT)$, and $a_k = a(kT)$. In eq.(5), state vector $\mathbf{x}_k$ is

$$\mathbf{x}_k = [\, r_k \;\; s_k \;\; a_k \,]^t, \tag{6}$$

state transition matrix $\mathbf{F}_a$ is

$$\mathbf{F}_a = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \tag{7}$$

and vector $\mathbf{g}_a$ is

$$\mathbf{g}_a = \begin{bmatrix} \frac{T^3}{3!} & \frac{T^2}{2} & T \end{bmatrix}^t. \tag{8}$$

Measured position of the target is denoted by y_k, and is assumed to be obtained by observation model

$$y_k = [\, 1 \quad 0 \quad 0 \,] \begin{bmatrix} r_k \\ s_k \\ a_k \end{bmatrix} + w_k, \tag{9}$$

where w_k is white Gaussian noise with 0 mean and variance σ^2.

2.2 Switching model

Candidate models to cope with model mismatching are defined here. We prepare models with different element of random walk, i.e., position, velocity, acceleration, jerk(difference of acceleration) and so on. They are as follows.

Firstly, position constant(random walk) model is defined by

$$r_k = r_{k-1} + Tv_k^{(r)} \tag{10}$$

where $v_k^{(r)} = v_r(kT)$ is white Gaussian system noise with 0 mean and variance ${\tau_r}^2$. Next, velocity constant(random walk) model is

$$\begin{bmatrix} r_k \\ s_k \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_{k-1} \\ s_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} v_k^{(s)} \tag{11}$$

where $v_k^{(s)} = v_s(kT)$ is white Gaussian system noise with 0 mean and variance ${\tau_s}^2$. Finally, jerk constant(random walk) model is

$$\begin{bmatrix} r_k \\ s_k \\ a_k \\ c_k \end{bmatrix} = \begin{bmatrix} 1 & T & \frac{T^2}{2} & \frac{T^3}{3!} \\ 0 & 1 & T & \frac{T^2}{2} \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{k-1} \\ s_{k-1} \\ a_{k-1} \\ c_{k-1} \end{bmatrix} + \begin{bmatrix} \frac{T^4}{4!} & \frac{T^3}{3!} & \frac{T^2}{2} & T \end{bmatrix}^t v_k^{(c)} \qquad (12)$$

where, $c_k = c(kT)$ stands for jerk, and $v_k^{(c)} = v_c(kT)$ is white Gaussian system noise with 0 mean and variance ${\tau_c}^2$.

Among these candidate models(eq. (5), (10), (11), and (12)), one must be selected as a system model. It depends on an indicator variable that specifies the selected model, and is denoted by i_k. It takes value equal to the highest order of the element, i.e., constant position model when $i_k = 1$, constant velocity model when $i_k = 2$, similarly for higher value $i_k = 3$ and $i_k = 4$ correspond to acceleration and jerk constant models respectively.

Switching of the indicator variable is according to Markov process. Transition matrix of the process (which consists of transition probability), is, for example, in case of four candidate models, with remaining probability to the same model $p_m = 0.95$,

$$\Pr\{i_k = i | i_{k-1} = j\} = \begin{bmatrix} 0.950 & 0.025 & 0.000 & 0.000 \\ 0.050 & 0.950 & 0.025 & 0.000 \\ 0.000 & 0.025 & 0.950 & 0.050 \\ 0.000 & 0.000 & 0.025 & 0.950 \end{bmatrix} \qquad (13)$$

where columns correspond to the indicator value before transit, and rows correspond to the after.

By augmenting state vector

$$\mathbf{z}_k = \left[\ (\mathbf{x}_k)^t \ \ i_k \ \right]^t, \qquad (14)$$

Bayesian switching model is written in a nonlinear state space model as follows

$$\mathbf{z}_k = \mathbf{F}(\mathbf{z}_{k-1} \ , \ \mathbf{v}_k) \qquad (15)$$

In eq.(15), state transition of indicator part i_k has already been shown as Markov process. State transition of ordinary part $\mathbf{x}_k$ is

$$\mathbf{x}_k = \mathbf{F}(i_k) \ \mathbf{x}_{k-1} + \mathbf{g}(i_k) \ \mathbf{v}_k, \qquad (16)$$

where $\mathbf{F}(i_k)$ is transition matrix that takes appropriate value depending on i_k, and the values come from eq.(5), (10), (11), and (12). In eq.(16), $\mathbf{v}_k$ is extended system noise vector defined by

$$\mathbf{v}_k = \left[\ v_k^{(r)} \ \ v_k^{(s)} \ \ v_k^{(a)} \ \ v_k^{(c)} \ \right]^t, \qquad (17)$$

and $\mathbf{g}(i_k)$ is defined by similar manner of $\mathbf{F}(i_k)$. Since $\mathbf{F}(i_k)$ and $\mathbf{g}(i_k)$ depend on an element i_k of state vector $\mathbf{z}_k$, eq.(16) is nonlinear. So the formula (15) is also nonlinear equation.

2.3 Heavy-tailed system noise

Due to the property of unknown maneuver, acceleration of the target may take 0, some constant value, and some gradual change. It may also take abrupt change among these values. It is represented by system noise term $v_k^{(a)}$ of eq.(17) that corresponds to the change of acceleration. In the conventional researches, Gaussian system noise is assumed to $v_k^{(a)}$. This causes dilemma to decide the variance of the system noise, i.e., to follow the abrupt change, variance must be large value, however, on the other hand, stability for constant acceleration period will be lost.

To satisfy both requirements to follow abrupt change and stability of constant period simultaneously, uni-modal heavy-tailed distribution is employed for the system noise. Where, uni-mode represents the small fluctuation with high probability for constant period, and heavy-tail allows abrupt change with small probability. We use Cauchy distribution for this in simulation study, since it is typical for such distribution. Probability density function of Cauchy distribution is shown in Fig.1 together with Gaussian distribution.

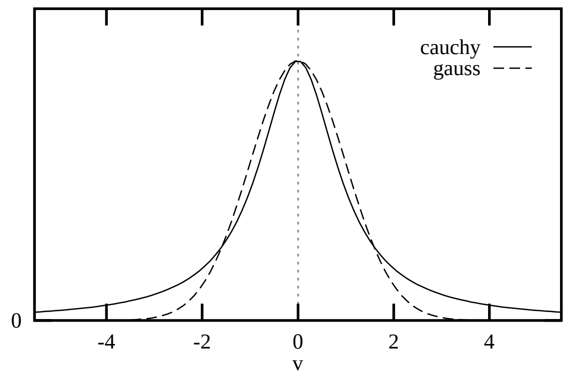

Figure 1: Cauchy distribution p.d.f. and Gaussian distribution p.d.f.

3 Particle filter

Particle filter is a generic term of a nonlinear non-Gaussian state estimation method using many particles in state space to approximate non-Gaussian distribution of the state. There are several researches separately developed in each field, Bootstrap filter [6] from the context of bootstrap method, Monte Carlo filter [12] from Monte Carlo simulation point of view, and Conditional density propagation(CONDENSATION) [9] in the field of computer vision. Their idea is common and they are considered as the special realization of sequential Monte Carlo method [13]. We employ Monte Carlo filter(MCF)[12], and its algorithm is shown as follows.

Let the observation series be denoted by

$$Y_N = \{y_1, y_2, \cdots, y_N\}. \tag{18}$$

The problem of state estimation is to calculate conditional distribution of the state by giving the observations. Its algorithm consists of iterative application of, one-step-ahead prediction procedure to obtain $p(\mathbf{z}_k|Y_{k-1})$, and filtering procedure to obtain $p(\mathbf{z}_k|Y_k)$, according to time order. There is also smoothing algorithm to obtain $p(\mathbf{z}_k|Y_{k+l})$, which is called fixed lag(l) smoothing.

3.1 State approximation by particles

In MCF, each conditional distribution of the state is approximated by many number of particles (realizations of the distribution). Filtering and smoothing procedures are computationally done by using these particles instead of analytical operation of distribution formula itself.

Let M be a number of particles. Notation of particles are as follows. For one-step-ahead prediction distribution,

$$\left\{\mathbf{p}_1^{(k)}, \mathbf{p}_2^{(k)}, \cdots, \mathbf{p}_M^{(k)}\right\} \sim p(\mathbf{z}_k|Y_{k-1}), \tag{19}$$

filtering distribution,

$$\left\{\mathbf{f}_1^{(k)}, \mathbf{f}_2^{(k)}, \cdots, \mathbf{f}_M^{(k)}\right\} \sim p(\mathbf{z}_k|Y_k), \tag{20}$$

and smoothing(with lag l) distribution

$$\left\{\mathbf{s}_1^{(k|k+l)}, \mathbf{s}_2^{(k|k+l)}, \cdots, \mathbf{s}_M^{(k|k+l)}\right\} \sim p(\mathbf{z}_k|Y_{k+l}). \tag{21}$$

3.2 Filtering algorithm

Initial distribution $p(\mathbf{z}_0|Y_0)$ is given and $\mathbf{f}_i^{(0)}$ $(i = 1, 2, \cdots, M)$ are calculated according to the distribution. Filtering algorithm is alternative application of two procedures according to the order of time index k, one-step-ahead prediction procedure to obtain particles $\mathbf{p}_i^{(k)}$ for $p(\mathbf{z}_k|Y_{k-1})$, and filtering procedure to obtain particles $\mathbf{f}_i^{(k)}$ for $p(\mathbf{z}_k|Y_k)$.

<u>One-step-ahead prediction procedure</u>:
Generate random vector $\mathbf{v}_i^{(k)}$ $(i = 1, 2, \cdots, M)$ according to the system noise distribution, and calculate particles by

$$\mathbf{p}_i^{(k)} = \mathbf{F}(\mathbf{f}_i^{(k-1)}, \mathbf{v}_i^{(k)}). \tag{22}$$

<u>Filtering procedure</u>:

Calculate likelihood of each particle $\mathbf{p}_i^{(k)}$ by

$$\alpha_i^{(k)} = p(y_k|\mathbf{p}_i^{(k)}), \tag{23}$$

where $p(y_k|\mathbf{p}_i^{(k)})$ can be obtained by using observation model. For example, in case of eq.(9), $p(y_k|\mathbf{p}_i^{(k)}) = r\left(y_k - r_i^{(k)}\right)$ with $r(\cdot)$ be probability density function of observation noise w_k and $r_i^{(k)}$ be r_k value of particle $\mathbf{p}_i^{(k)}$.

Resample particles according to

$$\mathbf{f}_i^{(k)} = \begin{cases} \mathbf{p}_1^{(k)} & \text{with prob.} \quad \alpha_1^{(k)} / \sum_{j=1}^{M} \alpha_j^{(k)} \\ \vdots & \quad\vdots \qquad\qquad \vdots \\ \mathbf{p}_M^{(k)} & \text{with prob.} \quad \alpha_M^{(k)} / \sum_{j=1}^{M} \alpha_j^{(k)} \end{cases} \tag{24}$$

3.3 Smoothing algorithm

Smoothing is carried out by augmenting the particle to have smoothing particles of past l times, and applying the filtering algorithm to the augmented particles. The augmented particles are

$$\mathbf{P}_i^{(k)} \equiv \left\{\mathbf{p}_i^{(k)}, \mathbf{s}_i^{(k-1|k-1)}, \mathbf{s}_i^{(k-2|k-1)}, \cdots, \mathbf{s}_i^{(k-l|k-1)}\right\}, \tag{25}$$

for one-step-ahead prediction, and

$$\mathbf{F}_i^{(k)} \equiv \left\{\mathbf{f}_i^{(k)}, \mathbf{s}_i^{(k-1|k)}, \mathbf{s}_i^{(k-2|k)}, \cdots, \mathbf{s}_i^{(k-l|k)}\right\}. \tag{26}$$

for filtering. Note that $\mathbf{f}_i^{(k)}$ is identical to $\mathbf{s}_i^{(k|k)}$.

3.4 Likelihood

Likelihood of the model to the observation series (18) can be approximately obtained by

$$\sum_{k=1}^{N} \log p(y_k|Y_{k-1}) \simeq \sum_{k=1}^{N} \log \left(\frac{1}{M} \sum_{j=1}^{M} \alpha_j^{(k)} \right). \quad (27)$$

Denoting eq.(27) by $l(\vartheta)$, ϑ is called "hyperparameter" of the model. In our model, it is a vector that consists of system noise variances (${\tau_r}^2$, ${\tau_s}^2$, ${\tau_a}^2$, and ${\tau_c}^2$), observation noise variance (σ^2), and remaining probability to the same model p_m in Markov process. They govern the performance of state estimation. The optimal value of hyperparameter, denoted by $\hat{\vartheta}$, is determined by maximizing the log-likelihood [1].

4 Simulation

One-dimensional trajectory generated by simulation is shown in Fig.2(a), where observation noise is according to $N(0, \sigma^2)$ and the variance is small as $\sigma^2 = 10^{-4}$. Acceleration of the trajectory is shown in Fig.2(b). In these figures, horizontal axis shows discrete time index k.

By applying Bayesian switching structure model to the trajectory. Both Gaussian model(ordinary) and Cauchy model(our proposal) are applied, and we have estimated position, velocity, acceleration, and jerk of the target. Number of particles is set to $M = 50,000$ in MCF, and system noise variances ${\tau_r}^2$, ${\tau_s}^2$, ${\tau_a}^2$, and ${\tau_c}^2$ are determined by grid search to maximize the likelihood.

Results, i.e. median of the estimated distribution of acceleration for both models are shown in Fig.3(a) and (b) by solid line, together with actual one by dashed line. We can seen that Cauchy model keeps stable at constant acceleration period without loss of following property of abrupt changing points.

For objective evaluation of results, errors between estimation and true for position, velocity, and acceleration are calculated. To avoid dependency of random number in MCF, we have run 100 times estimation with different random number seed. The average of mean root squares error for 100 runs are shown in Table.1. In the table, "win-rate" shows the rate that proposed model(Cauchy) scores better result than Gaussian model in 100 runs.

Results(ratio of appearance) of indicator variable for both models are shown in Fig.4(a) and (b). Indicator value 1(or 1 and 2) are majority for beginning part(i.e., constant position), 3 is major in the middle of the series, 2 is major at the ending part(constant velocity). We consider that the result is reasonable since the most appropriate model is majority at almost all the period of the series.

Table 1: Performance evaluation

	Cauchy	Gaussian	win-rate
position	5.97×10^{-3}	6.09×10^{-3}	75
velocity	5.42×10^{-3}	5.69×10^{-3}	87
accel	2.83×10^{-3}	2.97×10^{-3}	78

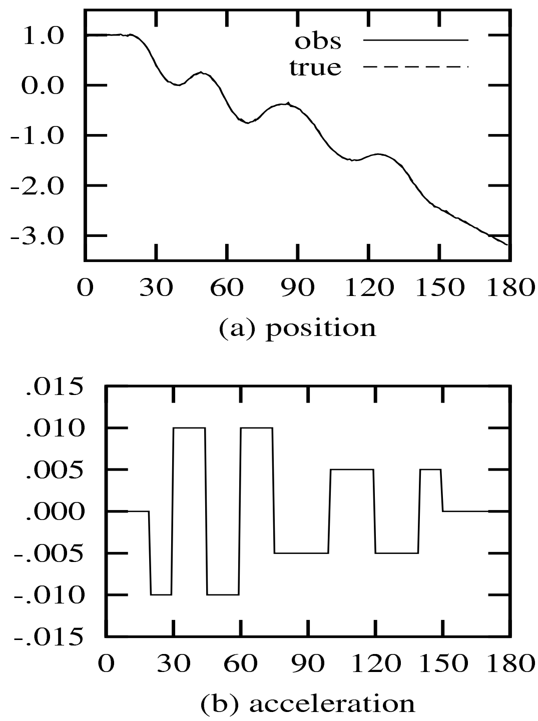

Figure 2: Trajectory of target.

5 Conclusion

We propose the use both of Bayesian switching structure(nonliner) and heavy-tailed uni-modal distribution(non-Gaussian) simultaneously in a target tracking problem. A simulation study of simple one-dimensional space tracking problem shows improvement of tracking performance by the proposed model compared with Gaussian system noise case.

For the future work, model extension to multi-dimensional position tracking is considered. By assuming radar observation of target position, observation model becomes nonlinear equation that transform Euclidean coordinate to polar coordinate. Application to real problem such as radar beam control also remains as a future work.

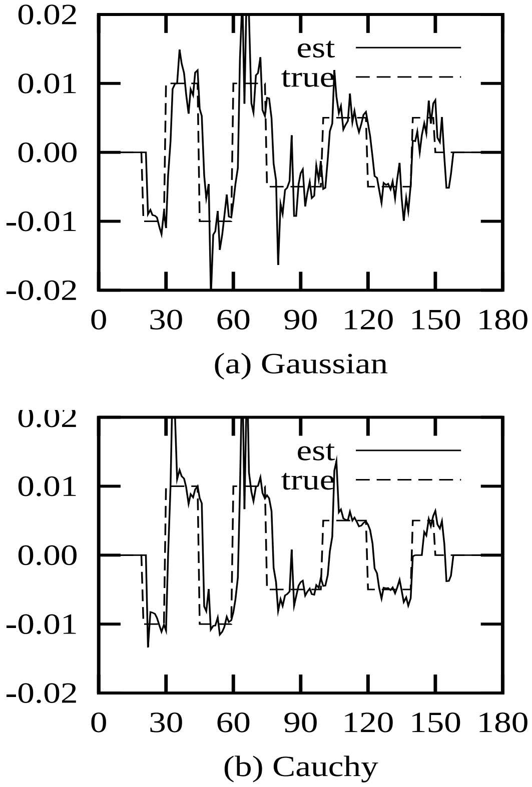

Figure 3: Result of acceleration.

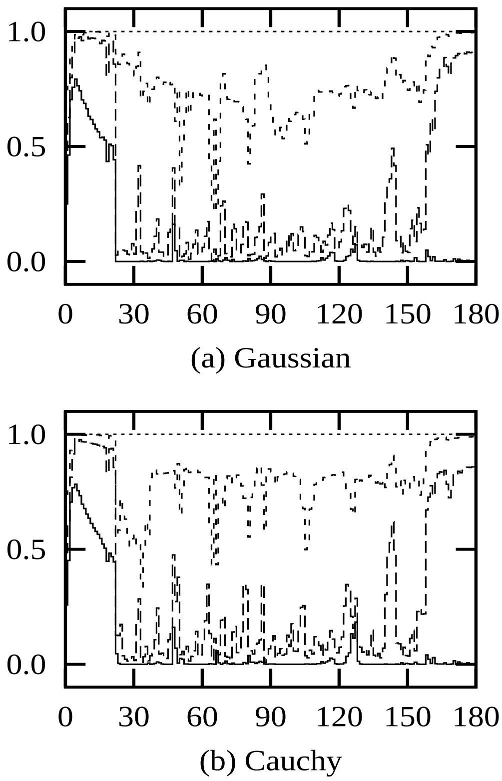

Figure 4: Result of accumulated ratio of models.

Acknowledgement: This study was carried out under the ISM Cooperative Research Program (2001-ISM·CPR-2024).

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