Tracking of feature points in dynamic image with classification into objects and 3D reconstruction by particle filters

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Abstract: A new model for tracking of feature points in dynamic image is proposed. The model is represented in a form of nonlinear state space model having state variables with positions of feature points, velocity of each object, and object labels that specify the associations between feature points and objects. We use particle filters with Rao-Blackwellization to estimate the state of the nonlinear model. By estimating the state, we obtain the tracking of feature points with velocity of each objects, and the classification of feature points into objects from the estimate of the associations. 3D reconstruction is also dealt with in this framework with camera projection in observation equation of the state space model. Experiments using real image for 2D tracking and 3D reconstruction show the efficiency of the model.
Key words: dynamic image, tracking, 3D reconstruction, classification, state space model, particle filters, Rao-Blackwellization.

1 Introduction

Structure from motion is one of the most interesting topic in the field of computer vision since it has wide applicability e.g., navigation of vehicle, manipulation of robot, etc. There is a bottom-up approach to this that uses feature points on the image sequence. This approach has an advantage that less assumption in the scene is required. There are several method in this category including one of the most famous method called factorization [17] that uses singular value decomposition(SVD) for measurement matrix to obtain two matrices corresponding to structure and motion. The original idea of the factorization has been proposed under assumptions of single motion and orthogonal projection, though, later, multiple motion extension [4] and extension to paraperspective projection [16] have been proposed.

Although factorization method is effective, it has one disadvantage for real application due to off-line property of SVD algorithm. Sequential method of the factorization has been proposed [15], however, it only deals with single motion in the scene. For realistic applications, sequential method for multiple motion is necessary. There is another approach to structure from motion [3] that uses state space model and extended Kalman filter(EKF) for state estimation, however it is also for single motion only due to the limitation of EKF.

Recently, new filtering techniques for nonlinear non-Gaussian state space model have been proposed and are called "particle filters". Particle filters use many number of particles in the state space to approximate the posterior distribution of the state given a series of observations. Recent development of computational performance of computer allows us to use this computer intensive method. Early works of particle filters are Bootstrap filter[8] and Monte Carlo filter[12]. Later, the ideas have been generalized into sequential estimation framework in [6] and [13]. Good survey of this topic is found in [7].

There are many applications of particle filters to dynamic image processing e.g., tracking of contour of target with spline curve[11], tracking of feature points in robust way with heavy-tailed distribution [9], feature points tracking and classification into objects in multiple objects scene [10], and 3D reconstruction from multiple feature points in multiple objects scene [14].

In this paper, we propose a general class of model for tracking of feature points in dynamic image with classification of the feature points into objects in the scene and recovery of 3D structure and motion. The model is represented in the form of nonlinear state space model for tracking feature points with discrete parameters, which we call "object label", and particle filters are effectively used to estimate the state of the model. Real image examples for 2D tracking and 3D reconstruction show the efficiency of the model.

2 Model

2.1 2D tracking and classification

Assume that there are p feature points and q objects in the scene of dynamic image.

Let $\mathbf{X}_j(k)$ be denoting position of j-th feature point at discrete time k in a form of row vector. Here we focus on 2D tracking problem on image plane, so the position is defined in $\Re^2$. Time evolution of the position is expressed by system equation

$$\mathbf{X}_j(k) = \mathbf{X}_j(k-1) + \mathbf{S}_{I_j(k)}(k-1), \tag{1}$$

where $\mathbf{S}_i(k-1)$ denotes velocity of i-th object at time

$k-1$, and $I_j(k)$ is an index variable taking integer value in $\{0,1,2,\cdots,q\}$. Note that $\mathbf{S}_0(k)$ is defined as zero vector. We refer to the index variable as "object label" since it specifies the object in which the feature point belongs to, except the case that the object label takes value 0.

Time evolution of velocity of i-th object is represented by system equation

$$\mathbf{S}_i(k) = \mathbf{S}_i(k-1) + \mathbf{v}_i(k), \quad \mathbf{v}_i(k) \sim N(\mathbf{0}, \mathbf{Q}), \quad (2)$$

where we refer to $\mathbf{v}_i(k)$ as system noise, and its covariance matrix $\mathbf{Q}$ is assumed to be diagonal with all diagonal elements identical to τ^2.

Object label is assumed to be according to Markov process

$$\begin{aligned} & Pr\{I_j(k) = n | I_j(k-1) = m\} \\ & \quad = \begin{cases} \rho & n = m \\ (1-\rho)/q & n \neq m \end{cases} \end{aligned} \quad (3)$$

where ρ is assumed to be high probability.

Observation process is defined so as to measure position of j-th feature point by

$$\mathbf{x}_j(k) = \mathbf{X}_j(k) + \mathbf{w}_j(k), \quad \mathbf{w}_j(k) \sim N(\mathbf{0}, \mathbf{R}), \quad (4)$$

where $\mathbf{x}_j(k)$ is the observed position of j-th feature point, and $\mathbf{w}_j(k)$ represents observation noise. Covariance matrix of the observation noise, $\mathbf{R}$, is assumed to be diagonal with all diagonal elements identical to σ^2.

Now the problem is to estimate positions of feature points,

$$\mathbf{X}_k \equiv (\mathbf{X}_1(k), \mathbf{X}_2(k), \cdots, \mathbf{X}_p(k))', \quad (5)$$

velocities of objects,

$$\mathbf{S}_k \equiv (\mathbf{S}_1(k), \mathbf{S}_2(k), \cdots, \mathbf{S}_q(k))', \quad (6)$$

and object labels,

$$\mathbf{I}_k \equiv (I_1(k), I_2(k), \cdots, I_p(k))', \quad (7)$$

from series of observed feature points,

$$\mathbf{x}_{1:k} \equiv (\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_k), \quad (8)$$

where

$$\mathbf{x}_k \equiv (\mathbf{x}_1(k), \mathbf{x}_2(k), \cdots, \mathbf{x}_p(k))'. \quad (9)$$

The notation in (8) will apply to other symbols.

2.2 3D reconstruction

Assume that the position of j-th feature point is defined in $\Re^3$ and denoted with its elements as

$$\mathbf{X}_j(k) = (X_j(k),\ Y_j(k),\ Z_j(k)), \quad (10)$$

where a constraint $Z_j(k) > 0$ is supposed since we consider a projection model. Observed position of j-th feature point is denoted with its elements as

$$\mathbf{x}_j(k) = (x_j(k),\ y_j(k)). \quad (11)$$

Then the model in the previous subsection for 2D tracking is extended to 3D reconstruction model from 2D observations by defining observation process as

$$\begin{bmatrix} x_j(k) \\ y_j(k) \end{bmatrix} = \begin{bmatrix} X_j(k)/Z_j(k) \\ Y_j(k)/Z_j(k) \end{bmatrix} + \begin{bmatrix} w_j^x(k) \\ w_j^y(k) \end{bmatrix}, \quad (12)$$

with $\mathbf{w}_j(k) \equiv \left(w_j^x(k) w_j^y(k)\right) \sim N(\mathbf{0}, \mathbf{R})$.

For the velocity of objects, we employ system equation in 3D space consisting of the same form of eq.(1) and (2). Object labels appearing in the equation is defined the same as in the previous subsection. Then, the problem here is the same as in the previous subsection, i.e., estimate position of feature points in 3D, $\mathbf{X}_k$, velocity of objects in 3D, $\mathbf{S}_k$, and object label, $\mathbf{I}_k$, from series of observed feature points in 2D, $\mathbf{x}_{1:k}$.

3 Estimation

To solve the problem of estimation in the previous section, we formulate a nonlinear state space model in a class of jump Markov model [5], and use particle filters for the state estimation.

3.1 State Space Representation

Let us form a state vector consisting of continuous variables of the model as

$$\mathbf{\Theta}_k \equiv (\mathbf{X}_k,\ \mathbf{S}_k). \quad (13)$$

We can formulate the model in a form of nonlinear state state representation with jump Markov structure

$$\begin{cases} \mathbf{\Theta}_k &= \mathbf{F}(\mathbf{I}_k)\mathbf{\Theta}_{k-1} + \mathbf{G}\mathbf{v}_k \\ \mathbf{x}_k &= \mathbf{H}(\mathbf{\Theta}_k) + \mathbf{w}_k \end{cases} \quad (14)$$

where $\mathbf{v}_k$ is system noise vector having elements $\mathbf{v}_1(k)$, $\mathbf{v}_2(k)$, $\cdots$, $\mathbf{v}_q(k)$, and $\mathbf{w}_k$ is observation noise vector having elements $\mathbf{w}_1(k)$, $\mathbf{w}_2(k)$, $\cdots$, $\mathbf{w}_p(k)$.

Although the model, eq.(14), has nonlinear factor in $\mathbf{H}(\cdot)$, extended Kalman filter(EKF) allows us to estimate the state when all the object labels up to current time k, denoted by $\mathbf{I}_{1:k}$, are given. That is, we can have $p(\mathbf{\Theta}_k|\mathbf{x}_{1:k}, \mathbf{I}_{1:k})$ approximately by EKF. However, the object labels, $\mathbf{I}_{1:k}$, are unknown in the actual situation, thus we need to estimate the object labels as well as the state.

To archive the simultaneous estimation of the state and the object label, we augment the state vector at time k into Ξ_k being defined by

$$\Xi_k \equiv (\mathbf{\Theta}_k,\ \mathbf{I}_k). \quad (15)$$

Then it is possible to rewrite the model in a form of conditional densities,

$$\begin{array}{lcl} \Xi_k & \sim & f(\,\cdot\,|\Xi_{k-1}\,;\tau^2,\rho), \\ \mathbf{x}_k & \sim & h(\,\cdot\,|\Xi_k\,;\sigma^2). \end{array} \tag{16}$$

In this formulation, the task of state estimation, which is called "filtering", is to obtain conditional distribution $p(\Xi_k|\mathbf{x}_{1:k})$ in recursive manner. Note that there is no closed form solution to the state estimation problem in this general formulation.

3.2 Particle Filters

Particle filters approximately solve the state estimation problem by using many number of weighted particles in the state space. The particles are drawn from a distribution called "proposal", and weight is assigned by ratio of the target distribution and the proposal distribution for each particle. Then the weighted particles approximately represent the target distribution in a manner of importance sampling. Particle filters modify the set of weighted particles when the new observation $\mathbf{x}_k$ becomes availiable, thus the algorithm is in recursive.

When condition suffices such that the particles are properly distributed in the state space, modification of only weights effectively update the distribution. However, the condition is not always satisfied, so in general, the update requires the modification of particles as well. We denote the set of weighted particles up to current time k by $\left\{\left(\Xi_{1:k}^{(l)},\omega_k^{(l)}\right)\right\}_{l=1}^{M}$, where l denotes particle number, and M is the number of particles. The algorithm of particle filters ensure that the set of weighted particles approximate the target distribution $p(\Xi_{1:k}|\mathbf{x}_{1:k})$.

The algorithm of particle filters is to obtain the set of weighted particles at time k by applying update procedure to the set of weighted particles at time $k-1$, and it can be divided into three steps, (1) draw of new particles, (2)weight update, and (3)re-sampling.

First, draw of new particles are proceeded as

$$\Xi_k^{(l)} \sim q(\cdot|\Xi_{1:k-1}^{(l)},\mathbf{x}_{1:k}) \tag{17}$$

for $l = 1,2,\cdots,M$. Where conditional distribution $q(\cdot|\cdot,\cdot)$ is called "proposal". We have some choice on the proposal within property

$$\begin{array}{l} \forall\Xi_k,\ p(\Xi_k|\Xi_{1:k-1},\mathbf{x}_{1:k}) > 0 \\ \qquad \Rightarrow q(\Xi_k|\Xi_{1:k-1},\mathbf{x}_{1:k}) > 0. \end{array} \tag{18}$$

Second, weight update proceeds as below for all $l = 1,2,\cdots,M$,

$$\omega_k^{(l)} \propto \omega_{k-1}^{(l)} \frac{f(\Xi_k^{(l)}\,|\Xi_{k-1}^{(l)}\,)h(\mathbf{x}_k\,|\Xi_k^{(l)}\,)}{q(\Xi_k^{(l)}|\Xi_{1:k-1}^{(l)},\mathbf{x}_{1:k})} \tag{19}$$

where weights are non-negative and normalized as sum to 1, i.e., $\sum_{l=1}^{M}\omega_k^{(l)} = 1$. It is based on the idea of importance sampling that any set of particles according proposal $q(\cdot)$ can be used to approximate the target distribution $p(\cdot)$ with weight $\omega(\cdot) = p(\cdot)/q(\cdot)$, where property $\forall x,\ p(x) > 0 \Rightarrow q(x) > 0$ holds. Eq.(19) is derived by using this idea to target distribution $p(\Xi_{1:k}|\mathbf{x}_{1:k})$ and proposal $q(\Xi_{1:k}|\mathbf{x}_{1:k})$ with re-use of past particles $\Xi_{1:k-1}^{(l)}$ to make algorithm be recursive.

Third and finally, re-sampling proceeds as follows; draw index variables $J(l)$ for $l = 1,2,\cdots,M$ according to the weights, i.e., index value $J(l) = i$ occurs with probability $\omega_k^{(i)}$. Then, replace all particles $l = 1,2,\cdots,M$ according to

$$\Xi_{1:k}^{(l)} := \Xi_{1:k}^{(J(l))} \tag{20}$$

and all weights are set to $1/M$.

Re-sampling is not necessarily applied for all times of filtering procedure. If the particles are properly distributed before the re-sampling, we can skip the re-sampling step. There are two major advantages to skip the re-sampling step; one is to allow parallel computation since other two steps can be parallelized while re-sampling step is not, and second is that Monte Carlo error involved in the draw of random number in the re-sampling step can be reduced thus variance of estimate is reduced by skipping re-sampling step.

3.3 Rao-Blackwellization

Further reduction of variance of the estimate is possible by using idea called "Rao-Blackwellization" [1]. The idea decompose the target distribution into analytical part and the rest as

$$p(\Xi_{1:k}|\mathbf{x}_{1:k}) = p(\mathbf{\Theta}_{1:k}|\mathbf{I}_{1:k},\mathbf{x}_{1:k})p(\mathbf{I}_{1:k}|\mathbf{x}_{1:k}). \tag{21}$$

Basically, filtering proceeds using particle filters for $p(\mathbf{I}_{1:k}|\mathbf{x}_{1:k})$ by noting the set of weighted particles $\left\{\left(\mathbf{I}_{1:k}^{(l)},\nu_k^{(l)}\right)\right\}_{l=1}^{M}$, and using EKF for analytical part $p(\mathbf{\Theta}_{1:k}|\mathbf{I}_{1:k}^{(l)},\mathbf{x}_{1:k}) \sim N(\bar{\mathbf{\Theta}}_{k|k}^{(l)},\Sigma_{k|k}^{(l)})$ with given particle $\mathbf{I}_{1:k}^{(l)}$.

Actual procedure consists of (1) draw of new particles, (2) EKF update, (3) weight update, and (4) re-sampling, as shown in the followings.

First, draw of new particles applies as

$$\mathbf{I}_k^{(l)} \sim q(\cdot|\mathbf{I}_{1:k-1}^{(l)},\mathbf{x}_{1:k}) \tag{22}$$

Second, EKF update proceeds then obtain

$$\left(\bar{\mathbf{\Theta}}_{k-1|k-1}^{(l)},\Sigma_{k-1|k-1}^{(l)}\right) \to \left(\bar{\mathbf{\Theta}}_{k|k}^{(l)},\Sigma_{k|k}^{(l)}\right) \tag{23}$$

where we use state space representation of eq.(14) with particle $\mathbf{I}_k^{(l)}$ instead of $\mathbf{I}_k$ in the equation. Third, weight update is done with

$$\nu_k^{(l)} \propto \nu_{k-1}^{(l)} \frac{p(\mathbf{I}_k^{(l)} \,|\mathbf{I}_{k-1}^{(l)}\,)p(\mathbf{x}_k \,|\mathbf{I}_k^{(l)}, \mathbf{x}_{1:k-1}\,)}{q(\mathbf{I}_k^{(l)}|\mathbf{I}_{1:k-1}^{(l)}, \mathbf{x}_{1:k})} \quad (24)$$

where $p(\mathbf{x}_k \,|\mathbf{I}_k^{(l)}, \mathbf{x}_{1:k-1}\,)$ has already been obtained at EKF update step. Finally, by applying the re-sampling step similar to the original particle filters, the procedure of Rao-Blackwellized particle filter completes. The difference of re-sampling step of Rao-Blackwelized version to the original one is that analytical part, $\left(\bar{\mathbf{\Theta}}_{k|k}^{(l)}, \Sigma_{k|k}^{(l)}\right)$, is also re-sampled together with $\mathbf{I}_{1:k}^{(l)}$.

3.4 Choice of proposal

It is important to choose a good proposal to make the particle filters efficient. Although optimal proposal is known, it is not practical in our model due to huge computational cost to enumerate all the combinations of object labels. So instead of using the optimal one, we propose to use sub-optimal proposal as follows;

Firstly, using a property of conditional independence for each feature point, we have

$$q(\mathbf{I}_k|\mathbf{I}_{1:k-1}^{(l)}, \mathbf{x}_{1:k}) = \prod_{j=1}^{p} q(I_j(k)|\mathbf{I}_{1:k-1}^{(l)}, \mathbf{x}_{1:k}). \quad (25)$$

Then, we employ sub-optimal proposal for each feature points as

$$\begin{aligned} &q(I_j(k)|\mathbf{I}_{1:k-1}^{(l)}, \mathbf{x}_{1:k}) \\ &\quad \propto p(\mathbf{x}_j(k)|\mathbf{x}_{1:k-1}, \mathbf{I}_{1:k-1}^{(l)}, I_j(k)), \end{aligned} \quad (26)$$

where, right hand side is exactly obtained by EKF in eq.(24), as a part of second term of the numerator.

Calculation of this exact proposal requires one-step-ahead prediction of the state in EKF and need to obtain mean vector and covariance matrix of observation depending on the one-step-ahead prediction. The computational cost of this calculation is high, so instead of using the exact one, we propose to use rough approximation of it by replacing its variance to be fixed and be large. Then what we need to calculate at the step of drawing from proposal is mean vector of observation depending only on the mean of one-step-ahead prediction of the state.

4 Experiment

2D tracking of feature points and 3D reconstruction from 2D feature points have been conducted.

4.1 2D Tracking

We have used dynamic image shown in figure 1, where two books are moving parallel to the image plane, and background does not move relative to the camera. Feature points are extracted in the 1st frame by corner detector [2], and searched around the position in previous frame for subsequent frames by using block matching based on normalized correlation. Feature points extracted by this procedure are shown with rectangles in the images of figure 1.

We have chosen four points for each of two books and background, and have applied our method to estimate object label and trajectory of feature points. For Rao-Blackwellized particle filter, $M = 50$k particles are used with conditions $\sigma^2 = 1$ and $\tau^2 = 0.1$. Estimation result of feature points' trajectory is shown in figure 2, where estimation result by EKF given the true object label is also plotted in the figure as ground truth of this experiment. Estimation result of object label is shown in table 1.

By looking at the result, close trajectory to the ground truth is obtained, while estimated object label is reasonable well.

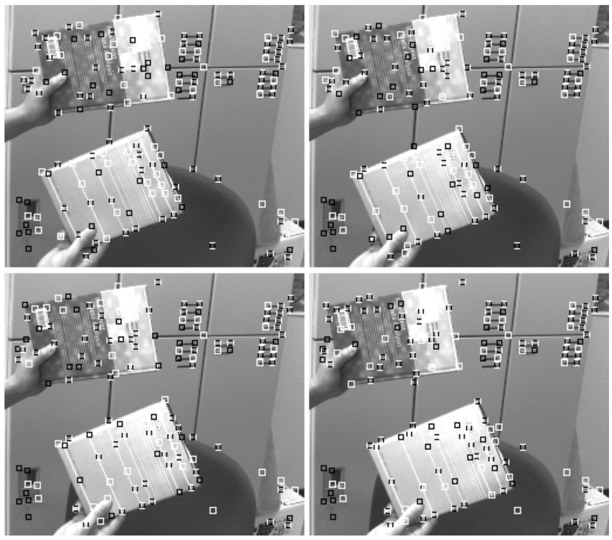

Figure 1: Image sequence and feature points for 2D tracking, 1st frame(upper left), 10th frame(upper right), 20th frame(lower left), and 30th frame(lower right).

4.2 3D Reconstruction

Dynamic image used for 3D reconstruction experiment is shown in figure 3. Two books are moving in the image, with motion including depth direction. Background does not move relative to the camera as same as the 2D experiment. The same procedures have been applied to the dynamic image, then we have obtained feature points shown with rectangles in the images.

Four feature points for each of two books and background are manually selected, then estimation of the proposed method has been proceeded. Rao-Blackwellized particle filter has been applied with conditions, $M =$

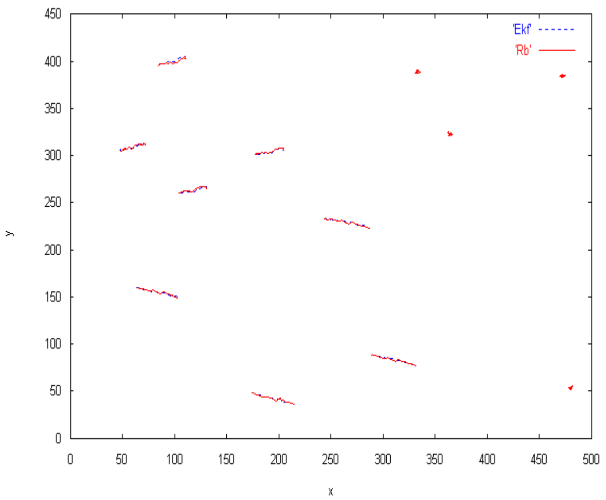

Figure 2: Estimation result of feature points for 2D tracking (Rb) together with EKF result of known object label (Ekf) as ground truth.

30k particles, $\sigma^2 = 1$, and $\tau^2 = 10^{-4}$. Estimation result of trajectory of feature points is shown in figure 4. As same as the 2D experiment case, estimation result by EKF given the true object label is also plotted in the figure as ground truth of this experiment. Estimation result of object label is shown in table 2.

By looking at the result, estimated trajectory is close enough to the ground truth, and estimated object label is reasonable well.

Figure 3: Image sequence and feature points for 3D reconstruction, 1st frame(upper left), 10th frame(upper right), 20th frame(lower left), and 30th frame(lower right).

Table 1: Estimated object label for 2D tracking.

k	Book-1				Book-2				Background			
1	0	0	2	1	1	1	1	2	2	2	2	0
2	0	0	2	0	1	1	1	2	2	2	2	2
3	0	0	2	2	1	1	1	1	2	2	2	2
4	0	0	0	0	1	1	1	1	2	2	2	2
5	0	0	0	0	1	1	1	1	2	2	2	2
6	0	0	0	0	1	1	1	1	2	2	2	2
7	0	0	0	0	1	1	1	1	2	2	2	2
8	0	0	0	0	1	1	1	1	2	2	2	2
9	0	0	0	0	1	1	1	0	2	2	2	2
10	0	0	0	0	1	1	1	2	2	2	2	2
11	0	0	0	0	1	1	1	2	2	2	2	2
12	0	0	0	0	1	1	1	1	2	2	0	2
13	0	0	0	2	1	1	1	1	2	2	0	2
14	0	0	0	0	1	1	1	1	2	2	0	2
15	0	0	0	0	1	1	1	1	2	2	2	2
16	0	0	0	0	1	1	1	1	2	2	1	2
17	0	0	0	0	1	1	1	1	2	2	1	2
18	0	0	0	0	1	1	1	1	2	2	1	2
19	0	0	0	0	1	1	1	1	2	2	1	2
20	0	0	0	0	1	1	1	1	2	2	2	2
21	0	0	2	0	1	1	1	1	2	2	2	2
22	0	0	0	2	1	1	1	1	2	2	2	2
23	0	0	0	2	1	1	1	1	2	2	2	2
24	0	0	0	0	1	1	1	1	2	2	2	2
25	0	0	0	0	1	1	2	1	2	2	2	2
26	0	0	0	0	1	1	2	1	2	2	2	2
27	0	0	0	0	1	1	1	1	2	2	2	2
28	0	0	0	0	1	1	1	1	2	2	2	2
29	0	0	1	0	1	1	1	1	2	2	2	2
30	0	0	0	0	1	1	1	1	2	2	2	2

5 Conclusion

We have proposed a general class of model for tracking of feature points in dynamic image with classification of the feature points into objects in the scene as well as 3D structure and motion recovery. The model is represented in a form of nonlinear state space model for tracking feature points with discrete parameters called object label. Particle filters with Rao-Blackwellization are effectively used to estimate the state of the model, with sub-optimal proposal using a property of conditional independence of each feature point in the model. Real image examples for 2D tracking and 3D reconstruction illustrate the efficiency of the model.

For the future works, there are several topics to investigate on the model. First, rotation has not been dealt with in the model, so it is interesting to involve it in the model. Second, current modeling for 3D reconstruction is limited to planar object parallel to the image plane in the scene. So more realistic modeling is necessary for real application. Third, to achieve online performance of the model, parallel computations such as using MPI or hardware implementation using e.g. FPGA are required. Application of this model to navigation of mobile robot is also interesting for the future work.

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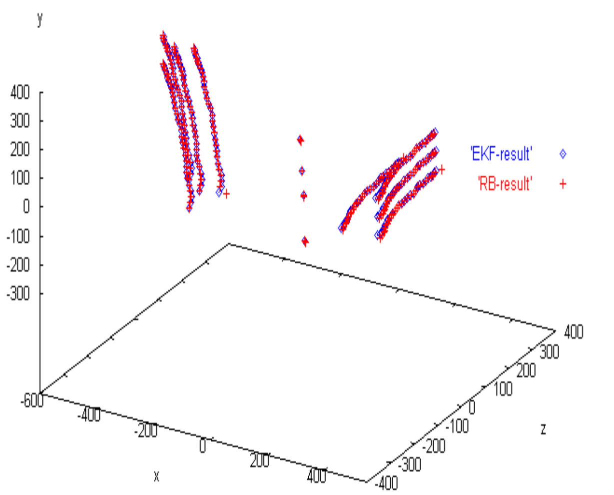

Figure 4: Estimation result of feature points for 3D reconstruction (RB-result) together with EKF result of known association (EKF-result) as ground truth.

Table 2: Estimated object label for 3D reconstruction.

k	Book-1				Book-2				Background			
1	0	0	0	0	1	1	2	2	2	2	2	2
2	0	0	0	2	1	1	2	2	2	2	2	2
3	0	0	0	0	1	1	1	1	2	2	2	2
4	0	0	0	0	1	1	1	1	2	2	2	2
5	0	0	0	0	1	1	1	1	2	2	2	2
6	0	0	0	0	1	1	1	1	2	2	2	2
7	0	0	0	0	1	1	1	1	2	2	2	2
8	0	0	0	0	1	1	1	1	2	2	2	2
9	0	0	0	0	1	1	1	1	2	2	2	2
10	0	0	0	0	1	1	1	1	2	2	2	2
11	0	0	0	0	1	1	1	1	2	2	2	2
12	0	0	0	0	1	1	1	1	2	2	2	2
13	0	0	0	0	1	1	1	1	2	2	2	2
14	0	0	0	0	1	1	1	1	2	2	2	2
15	0	0	0	0	1	1	1	1	2	2	2	2
16	0	0	0	0	1	1	1	1	2	2	2	2
17	0	0	0	0	1	1	1	1	2	2	2	2
18	0	0	0	0	1	1	1	1	2	2	2	2
19	0	0	0	0	1	1	1	1	2	2	2	2
20	0	0	0	0	1	1	1	1	2	2	2	2
21	0	0	0	0	1	1	1	1	2	2	2	2
22	0	0	0	0	1	1	1	1	2	2	2	2
23	0	0	0	0	1	1	1	1	2	2	2	2
24	0	0	0	0	1	1	1	1	2	2	2	2
25	0	0	0	0	1	1	1	1	2	2	2	2
26	0	0	0	0	1	1	1	1	2	2	2	2
27	0	0	0	0	1	1	1	1	2	2	2	2
28	0	0	0	0	1	1	1	1	2	2	2	2
29	0	0	0	0	1	1	1	1	2	2	2	2
30	0	0	0	0	1	1	1	1	2	2	2	2

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