MANEUVERING TARGET TRACKING BY NONLINEAR NON-GAUSSIAN STATE SPACE MODEL

NORIKAZU IKOMA*, NAOYUKI ICHIMURA†, AND TOMOYUKI HIGUCHI‡

* *Kyushu Institute of Technology, Kita-Kyushu, Fukuoka 804-8550, Japan,* ikoma@comp.kyutech.ac.jp
† *Electrotechnical Laboratory, Tsukuba, Ibaraki 305-8568, Japan,* ichimura@etl.go.jp
‡ *The Institute of Statistical Mathematics, Minato-ku, Tokyo 106-8569, Japan,* higuchi@ism.ac.jp

Abstract. The aim of this research is tracking of maneuvering target such as ship, aircraft, and so on. Position of the target is measured by radar in polar coordinate. Dynamics of target is described by system model with state vector represented in Cartesian coordinate, and measurement process is formulated by observation model with observation vector represented in polar coordinate. Since polar coordinate is used in observation vector, the observation model becomes nonlinear with respect to the state vector. To follow abrupt changes of the target's motion due to sudden operation of acceleration pedal, break, and steering, we propose a use of heavy-tailed non-Gaussian distribution for system noise. Using Monte Carlo filter, which is a kind of sequential Monte Carlo method, we can estimate the target's state. Usefulness of the method is shown by simulation.

Key words and phrases: nonlinear, non-Gaussian, heavy-tailed distribution, state space model, target tracking

1. Model

Dynamics of maneuvering target is firstly described by continuous time model with Gaussian noise, and discretizing it with respect to time, we obtain a discrete time model. In the continuous time model, dynamics of acceleration of the target is denoted by $\dot{a}(t) = -\alpha a(t) + u(t)$, where $a(t)$ is the acceleration, $\alpha < 1$ is damping coefficient, and $u(t)$ is Gaussian white noise. For more detail, see Ohsumi and Yasui(2000).

1.1 *System model*

Let Δt is sampling time of discretization. Discrete time points $t = t_0 + k\Delta t$ for $k = 0, 1, 2, \cdots$ are treated. Then ,the discretized model is

$$\mathbf{x}_k = \mathbf{A}\ \mathbf{x}_{k-1} + \mathbf{B}\ \mathbf{v}_k, \tag{1.1}$$

where $\mathbf{x}_k = [r_x(k), r_y(k), s_x(k), s_y(k), a_x(k), a_y(k)]^T$ is state vector, where $r_x(k)$,$r_y(k)$ are positions, $s_x(k)$,$s_y(k)$ are velocities, and $a_x(k)$,$a_y(k)$ are acceleration, in Cartesian coordinate respectively. In eq.(1.1),

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & \Delta t & 0 & a_1 & 0 \\ 0 & 1 & 0 & \Delta t & 0 & a_1 \\ 0 & 0 & 1 & 0 & a_2 & 0 \\ 0 & 0 & 0 & 1 & 0 & a_2 \\ 0 & 0 & 0 & 0 & e^{-\alpha\Delta t} & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{-\alpha\Delta t} \end{bmatrix} \tag{1.2}$$

is state transition matrix,

$$\mathbf{B} = \begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & b_1 & 0 & b_2 & 0 & b_3 \end{bmatrix}^{\mathrm{T}} \tag{1.3}$$

is a matrix for observation noise, and $\mathbf{v}_k = [v_x(k), v_y(k)]^T$ is observation noise vector. The entities of eq.(1.3) are as follows;

$$b_1 = \frac{1}{\alpha}\left(\frac{(\Delta t)^2}{2} - a_1\right), \tag{1.4}$$

$$a_1 = b_2 = \frac{1}{\alpha}(\Delta t - a_2), \tag{1.5}$$

$$a_2 = b_3 = \frac{1}{\alpha}\left(1 - e^{-\alpha\Delta t}\right). \tag{1.6}$$

As a result of discretization of Gaussian white system noise of continuous model, we obtain a Gaussian white distribution for system noise vector $\mathbf{v}_k$ with each component independent. Here, to track the maneuvering target with abrupt change of its acceleration, we propose to use heavy-tailed non-Gaussian distribution as follows;

$$\mathbf{v}_k \sim C(\mathbf{0}, \mathbf{Q}_c), \quad \mathbf{Q}_c = \begin{bmatrix} {q_x}^2 & 0 \\ 0 & {q_y}^2 \end{bmatrix}, \tag{1.7}$$

where, $\mathbf{C}$ denotes heavy-tailed distribution with central position $\mathbf{0}$ and dispersion $\mathbf{Q}_c$. Cauchy distribution is typical as the heavy-tailed distribution. In scalar case, Probability density function of Cauchy distribution is given by

$$p_c(v) = q / \left\{\pi(v^2 + q^2)\right\} \tag{1.8}$$

where central position is 0 and dispersion is controlled by q. It has relatively high probability for large $|v|$ values compared with Gaussian distribution.

1.2 *Observation model*

Measurement process of target position by radar is written in observation model

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{w}_k, \tag{1.9}$$

where $\mathbf{y}_k = [y_\theta(k), y_g(k)]^T$ is observation vector that consists of bearing $y_\theta(k)$ and range $y_g(k)$,

$$\mathbf{h}(\mathbf{x}_k) = \begin{bmatrix} \tan^{-1}\left\{\dfrac{r_x(k)}{r_y(k)}\right\} \\ \sqrt{r_x(k)^2 + r_y(k)^2} \end{bmatrix} \tag{1.10}$$

denotes nonlinear characteristic of measurement process by radar, and

$$\mathbf{w}_k = [w_\theta(k), w_g(k)]^T \tag{1.11}$$

shows observation noise vector, where

$$\begin{bmatrix} w_\theta(k) \\ w_g(k) \end{bmatrix} \sim N(\mathbf{0}, \mathbf{R}), \quad \mathbf{R} = \begin{bmatrix} \sigma_\theta^2 & 0 \\ 0 & \sigma_g^2 \end{bmatrix}. \tag{1.12}$$

2. State estimation

For state estimation of nonlinear non-Gaussian model described above, we have employed Monte Carlo Filter(MCF), which is a kind of Sequential Monte Carlo method(Liu and Chen(1998)). There are several similar method like bootstrap filter by Gordon et al.(1993), and CONDENSATION by Isard and Blake(1998). For the detail of MCF, see Kitagawa(1996).

3. Simulation

Synthetic data have been generated to simulate the maneuvering target(ship) as the same trajectory to Ohsumi and Yasui(2000), and they are shown in Fig.1 in Cartesian coordinate. The polar coordinates are actually observed and used for state estimation. Observation noise is small in this experiment, i.e., variance of bearing is 10^{-10} , and that of range is 10^{-2}.

Gaussian and non-Gaussian model are applied with extended Kalman filter(EKF) and MCF, respectively. We assume the variances of observation noise vector are known. System noises are determined by log-likelihood for both models. The number of particles of MCF is $100,000$.

Estimation result of velocity and acceleration by Gaussian model with EKF and non-Gaussian model with MCF are shown in Fig.2 and Fig.3 . In these figures, solid lines show the estimation result by MCF, long-dashed lines show the etimation result by EKF, and short-dashed lines show the true trajectory. The result of EKF is obtained from mean vector of filter distribution(Gaussian), and the one of MCF is the median of marginal distribution, computed by sorting the particles of filter distribution. We can see that EKF-Gaussian results delayed response compared with MCF-Cauchy estimates, especially in acceleration of x-axis.

Acknowledgement This study was carried out under the ISM Cooperative Research Program (2000-ISM·CPR-1005).

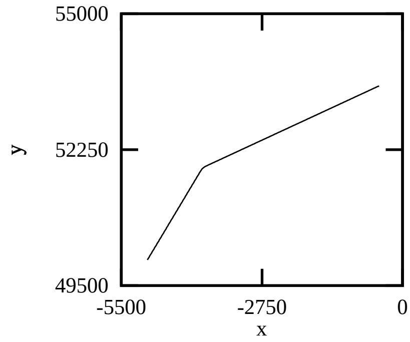

Fig. 1. Trajectory

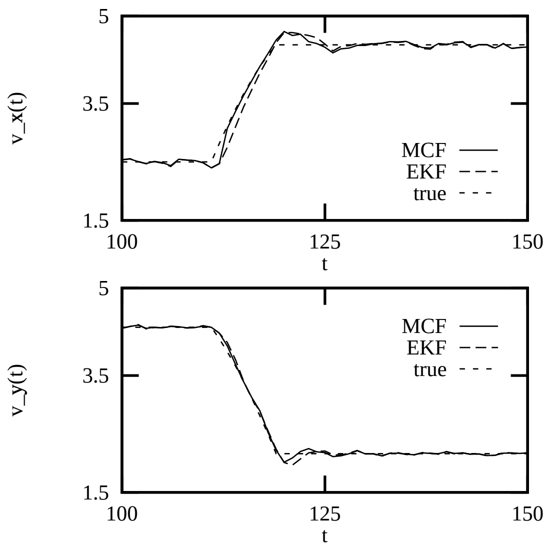

Fig. 2. Estimation results, $\hat{\mathbf{v}}(k|k)$

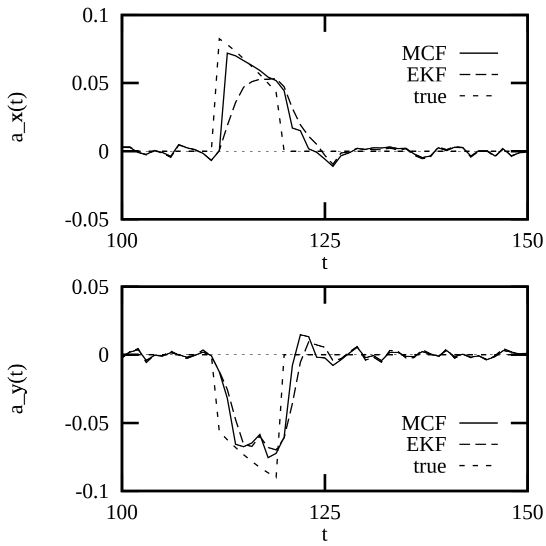

Fig. 3. Estimation results, $\hat{\mathbf{a}}(k|k)$

References

Gordon, N.J., Salmond, D.J., and Smith, A.F.M. (1993). Novel approach to nonlinear/non-Gaussian Bayesian State Estimation, *IEE Proceedings-F*, **140**, No.2, 107-113.

Isard, M. and Blake, A. (1998). CONDENSATION – Conditional Density Propagation for Visual Tracking, *Journal of Computer Vision*, **29**, No.1, 5-28.

Kitagawa, G. (1996). Monte Carlo filter and smoother for non-Gaussian nonlinear state space models, *Journal of Computational and Graphical Statistics* **5**,No.1,1-25.

Liu,J.S. and Chen,R.(1998). Sequential Monte Carlo methods for dynamic systems, *Journal of the American Statistical Association*, **93**, 1032-1044.

Ohsumi,A. and Yasui,S.(2000). Tracking of a maneuvering target considering its kinematic constrains, Proc.of world automation congress, 3rd intl.Sympo.on Intelligent automat.& contr.