Estimation of Time Varying Peak of Power Spectrum based on Non-Gaussian Nonlinear State Space Modeling

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Abstract

An estimation method of time varying peak frequencies of power spectrum is proposed. The method is based on a new nonlinear model that has peak frequencies of power spectrum as time varying parameters. Estimation of time varying peaks is done with the aid of non-Gaussian filtering method. Time invariant parameters contained in the model is numerically obtained by maximizing the likelihood function. The order of the model is determined by AIC (Akaike Information Criterion). The model selection among several models is also done by AIC. Model extension to have time invariant factors of power spectrum has been done. Numerical experiments with artificially generated data are shown at the end of this paper.

1 Introduction

A new model that has time varying peaks of power spectrum has been proposed. It has been named TVPP(Time Varying Peaks of Power spectrum) model. The key idea of the model is the use of time smoothness of spectrum peaks instead of AR coefficients, which are used in conventional models e.g. [5, 6]. With the aid of recent researches of non-Gaussian filtering method [7, 8], the proposed model can be applied to practical problems. The advantages of proposed model are as follows; 1) smooth evolution of power spectral peaks can directly be estimated, 2) confidential interval of time varying peak is obtained, and 3) decomposition of spectrum into time varying peaks and time invariant factors is available.

Spectral analysis is widely used in many part of research field. The aim of spectral analysis is to know the cyclical features of target object through the spectrum. Peak frequency of power spectrum is one of the most important factors of cyclical features. The information of peak frequency can be used for the design of vibrative safety system including the target object. In time domain analysis of spectrum, statistical models are generally used to estimate the power spectrum. AR (Auto-Regressive) model is one of the most simple and important models for the time domain analysis. In AR model, power spectrum can be obtained through estimating AR coefficients. Smooth shape with several peaks is a feature of spectrum obtained by AR model.

There are many targets that have nonstationary spectrum, for example, seismic wave, vibration with nonstationary condition, and human voice. Since stationarity is assumed in AR model, the model is not effective for nonstationary series. In recent researches, statistical models to obtain nonstationary spectrum have been proposed. Locally stationary AR model [10, 4] consists of stationary AR models applied to each short interval of time series. TVCAR (Time Varying Coefficient AR) model [5, 6] contains AR coefficients that have time changes. In both models, nonstationary power spectrum can be obtained through estimating AR coefficients changing with time.

In these conventional models, the number of parameters are usually greater than the number of data. Generally, this is ill condition to estimate the parameters. By imposing a time smoothness to each AR coefficient, estimation of the model becomes possible. Here, peaks of power spectrum are important information, however, smooth changes of them are not assumed in the conventional researches. In the conventional models, the smoothness is assumed in only AR coefficients. The smoothness of AR coefficients merely means smooth changes of power spectrum for all frequencies.

Since peak frequency of power spectrum is an important feature in nonstationary analysis, it seems reasonable to define the model with smooth change of peak frequencies rather than AR coefficients. However, this definition involves nonlinear factor in the model. Because of many difficulties of nonlinear models, this kind of model has not been investigated so far. Simple applications of Kalman filtering algorithm to nonlinear models generally fail. The use of extended Kalman filtering, which uses first order approximation of nonlinear formula, is still limited.

On the other hand, non-Gaussian filtering methods have been developed recently. The key idea of non-Gaussian filtering is an approximation of non-Gaussian distributions, for example, the use of weighted sum of Gaussian distributions [1], numerical approximation called non-Gaussian nonlinear state space modeling method [7, 8], and the use of sample according to the distribution [9]. With the aid of non-Gaussian filtering method, the difficulties of nonlinear model are overcame. Therefore, the proposed model is practical under the current situation of research field.

In this paper, we propose a new model that has peak frequencies of power spectrum as time varying parameters. Non-Gaussian nonlinear state space modeling method [7, 8] is applied to the model estimation. Time invariant parameters included in the model can be determined by maximizing the likelihood function. Order determination of the model can be done by minimum AIC (Akaike Information Criterion) procedure [2, 3]. Model selection among several models is also done by the minimum AIC procedure. An extension of the original model to have time invariant factor of power spectrum is also proposed. In this extension, we have introduced a stationary AR model to construct a multiplicative model with original model. Coefficients of the stationary AR model are estimated in numerical optimization process together with the other time invariant parameters.

Numerical experiments by using artificially generated data have been done to show the efficiency of the model. Three data sets have been applied, they are; (1) one time varying peak of power spectrum, (2) two time varying peaks of power spectrum, and (3) one time varying peak and one time invariant peak of power spectrum. We have compared the results of our model with that of TVCAR model. Our model scores better result than TVCAR model, with respect to AIC, expected log-likelihood, and mean squares error of peak frequency.

2 Time Varying Peaks of Power Spectrum Model(TVPP model)

2.1 Stationary AR model

Let us define an AR(Auto-Regressive) model of order p as follows,

$$y_t = \sum_{j=1}^{p} a_j y_{t-j} + \varepsilon_t, \quad t = p+1 \sim N. \tag{1}$$

Where $Y_N = \{y_1, y_2, \ldots, y_N\}$ is a data set, $\{a_t | t = 1, 2, \ldots, p\}$ is a set of AR coefficients, and $\{\varepsilon_t\}$ is a Gaussian white noise with mean 0 and variance σ^2. The power spectrum, which is theoretically obtained from AR model, is as follows,

$$p(w) = \frac{\sigma^2}{\left|1 - \sum_{j=1}^{p} a_j e^{-ijw}\right|^2}, \tag{2}$$

where $w \in [0, \pi]$ denotes angular frequency.

The characteristic equation of AR model is given by,

$$1 - \sum_{j=1}^{p} a_j B^j = 0, \tag{3}$$

where B is the backward shift operator such that $By_t = y_{t-1}$.

Assume that the order of AR model p is even, i.e. $p = 2m$, and all solutions of characteristic equation are complex. Conjugate pairs of solution are denoted by $r_k e^{-i\theta_k}$ and $r_k e^{i\theta_k}$, for $k = 1, 2, \ldots, m$.

Stationarity condition of AR model is that all solutions are located at the outside of unit circle, i.e., $r_k > 1$ for $k = 1, 2, \ldots, m$.

We can rewrite the characteristic equation (3) by using its solutions as follows,

$$\prod_{k=1}^{m} (1 - r_k e^{-i\theta_k} B)(1 - r_k e^{i\theta_k} B) = 0. \tag{4}$$

Let w_k be a peak frequency of power spectrum (2), we can easily see that the denominator of (2) is locally minimized at the point of w_k.

For all $k = 1, 2, \ldots, m$, by assuming $r_k \simeq 1$ under the stationarity condition $r_k > 1$, θ_k can be an approximation of peak frequency ω_k of power spectrum (2).

We will use this approximation to derive a new model to estimate the time varying peak frequencies of nonstationary power spectrum in the following section.

2.2 Original Model

We propose a new model to estimate time varying peak frequencies of power spectrum (TVPP model) as follows,

$$\mathbf{\Theta}_t = \mathbf{\Theta}_{t-1} + \mathbf{w}_t, \tag{5}$$

$$y_t = \sum_{j=1}^{p} a_j^{(p)}(\mathbf{\Theta}_t, \mathbf{R}) y_{t-j} + \varepsilon_t, \quad p = 2m. \tag{6}$$

Where, $\boldsymbol{\Theta}_t$ is a vector of peak frequencies of time t such that

$$\boldsymbol{\Theta}_t = [\theta_1(t), \theta_2(t), \ldots, \theta_m(t)]^T, \tag{7}$$

and $\mathbf{R}$ is a vector as follows,

$$\mathbf{R} = [r_1, r_2, \ldots, r_m]^T. \tag{8}$$

$\mathbf{w}_t$ is a random vector that specifies the smoothness of peak frequencies $\boldsymbol{\Theta}_t$, and it is denoted as follows,

$$\mathbf{w}_t = [w_1(t), w_2(t), \ldots, w_m(t)]^T, \;\; \mathbf{w}_t \sim N(0, Q), \tag{9}$$

where Q is the covariance matrix such that

$$Q = \mathrm{diag}\left(\tau_1^2, \tau_2^2, \ldots, \tau_m^2\right). \tag{10}$$

We can see in (6) that AR coefficients are parametrized by $\boldsymbol{\Theta}_t$ and $\mathbf{R}$. The relationship between AR coefficients and these parameters is derived by solving the following equations,

$$\begin{cases} 1 - \sum_{j=1}^{p} a_j B^j = 0 \\ \prod_{k=1}^{m} (1 - r_k e^{i\theta_k(t)} B)(1 - r_k e^{-i\theta_k(t)} B) = 0. \end{cases} \tag{11}$$

The examples of coefficients $a_j^{(p)}(\cdot, \cdot)$, in case of $p = 2$ and 4 are shown as follows.

In case of $p = 2$:

$$\begin{aligned} a_1^{(2)}(\boldsymbol{\Theta}_t, \mathbf{R}) &= 2r\cos\theta_t \\ a_2^{(2)}(\boldsymbol{\Theta}_t, \mathbf{R}) &= -r^2 \end{aligned} \tag{12}$$

where, we simply denote $\theta_1(t)$ and r_1 by θ_t and r.

In case of $p = 4$:

$$\begin{aligned} a_1^{(4)}(\boldsymbol{\Theta}_t, \mathbf{R}) &= 2\left(r_1\cos\theta_1(t) + r_2\cos\theta_2(t)\right) \\ a_2^{(4)}(\boldsymbol{\Theta}_t, \mathbf{R}) &= -r_1^2 - r_2^2 - 4r_1r_2\cos\theta_1(t)\cos\theta_2(t) \\ a_3^{(4)}(\boldsymbol{\Theta}_t, \mathbf{R}) &= 2r_1r_2\left(r_2\cos\theta_1(t) + r_1\cos\theta_2(t)\right) \\ a_4^{(4)}(\boldsymbol{\Theta}_t, \mathbf{R}) &= -r_1^2r_2^2 \end{aligned} \tag{13}$$

Since the stationarity condition of AR model is required, it should be kept $r_k > 1$ for $k = 1, 2, \ldots, m$. When $r_k \simeq 1$ is hold, $\theta_k(t)$ is approximately a peak frequency of power spectrum.

Our model still has the other parameters which are time invariant. They are denoted by vector form as follows,

$$\mathbf{x} = \left[\sigma^2, \tau_1^2, \tau_2^2, \ldots, \tau_m^2, r_1, r_2, \ldots, r_m\right]^T. \tag{14}$$

3 Estimation Method

Since nonlinear equation is involved in the proposed model, non-Gaussian filtering is required for the model estimation. There are several methods for non-Gaussian filtering according to the approximation method of non-Gaussian distributions [1],[7, 8],[9]. In this paper, we have employed non-Gaussian nonlinear state space modeling method [7, 8], and it is summarized in this section. The key idea of the method [7, 8] is an approximation of non-Gaussian distributions by step-wise function or partially linear function. For the convenience, we simply explain in case of the dimension of $\mathbf{\Theta}_t$ is 1 and use the approximation by step-wise function in the following text. The complete algorithm involving estimation and order determination is written in **Appendix A** by Pascal(computer language) like style.

3.1 Numerical Approximation

Non-Gaussian distributions are approximated by step-wise function in the non-Gaussian method [7, 8]. Since the step-wise function can be identified by certain number of numerical points, the non-Gaussian distributions can be denoted by numerical points. Using these numerical points, estimation process that involves integrations can be done numerically.

In the estimation process, three kind of estimations appear depending on the time of given series. One step ahead prediction, denoted by $p(\mathbf{\Theta}_{t+1}|Y_t)$, is the future estimation by using the data up to current time t. Filtering, denoted by $p(\mathbf{\Theta}_t|Y_t)$, is the current estimation by using the data up to current time t. Smoothing, denoted by $p(\mathbf{\Theta}_t|Y_N)$, is the estimation of past time t by using all available data up to current time N.

These distributions are numerically approximated as follows. Since our attention is paid to frequency domain such that $[0, \pi]$, it is enough to make numerical approximation on this domain. Divide $[0, \pi]$ into M adjoint intervals, and a set of boundary points are denoted by

$$\mathbf{b} = \{b_0, b_1, b_2, \ldots, b_M\}, \tag{15}$$

where $b_{i-1} < b_i$ for all $i = 1, 2, \ldots, M$.

Filtering, one-step-ahead prediction, and smoothing distributions are represented by using numerical points corresponding to the above intervals. The notations of them are as follows;

Filtering distribution $p(\theta_t|Y_t)$ is denoted by

$$f_t = \{f_1(t), f_2(t), \ldots, f_M(t)\}^T, \tag{16}$$

one-step-ahead prediction distribution $p(\theta_t|Y_{t-1})$ is denoted by

$$p_t = \{p_1(t), p_2(t), \ldots, p_M(t)\}^T, \tag{17}$$

and smoothing distribution $p(\theta_t|Y_N)$ is denoted by

$$s_t = \{s_1(t), s_2(t), \ldots, s_M(t)\}^T. \tag{18}$$

The distribution of w_t is also represented by numerical points for the convenience of calculation. Since $w_t = \theta_t - \theta_{t-1}$, the values of distribution corresponding to $b_i - b_j$ for $i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, M$ are stored as the numerical points as follows,

$$q = \{q_{ij}|i = 1, 2, \ldots, M, \ j = 1, 2, \ldots, M\}. \tag{19}$$

3.2 Prediction and Filtering

Starting from initial distribution $p(\theta_0)$, by applying prediction and filtering process alternatively, we can obtain the prediction and the filtering distributions for $t = 1, 2, \ldots, N$. Since the process is going toward the time direction, this can be called forward phase of the estimation process.

Exact calculation of one-step-ahead prediction distribution is derived as follows,

$$p(\theta_t|Y_{t-1}) = \int p(\theta_t|\theta_{t-1})p(\theta_{t-1}|Y_{t-1})d\theta_{t-1}. \tag{20}$$

The numerical implementation of one-step-ahead prediction becomes as follows,

$$p_i(t) = \sum_{j=0}^{M-1} (b_{j+1} - b_j) \cdot f_j(t-1) \cdot q_{ij}. \tag{21}$$

Exact calculation of filtering distribution is obtained from prediction distribution as follows,

$$p(\theta_t|Y_t) = \frac{p(\theta_t|Y_{t-1})p(y_t|\theta_t, Y_{t-1})}{p(y_t|Y_{t-1})}, \tag{22}$$

where $p(y_t|Y_{t-1})$ is calculated by integrating the numerator:

$$p(y_t|Y_{t-1}) = \int p(\theta_t|Y_{t-1})p(y_t|\theta_t, Y_{t-1})d\theta_t. \tag{23}$$

The numerical implementation of filtering becomes as follows,

$$f_i(t) = f'_i(t) / \sum_{j=0}^{M-1} (b_{j+1} - b_j) \cdot f'_j(t-1), \tag{24}$$

$$f'_i(t) = p_i(t) \cdot r \left(y_t - \sum_{j=1}^{2} a_j(b_i) \cdot y_{t-j} \right). \tag{25}$$

Where $r(\cdot)$ is the *pdf* for ε_t. Although it is assumed in our model that $r(\cdot)$ is a Gaussian distribution, it is not necessary in general.

3.3 Parameter Estimation and Order Determination

The time invariant parameters involved in the proposed model are estimated by maximizing the likelihood function. The log-likelihood function is used for the maximization instead of likelihood function, and it is shown as follows,

$$l(\mathbf{x}) = \sum_{t=1}^{N} \log p(y_t|Y_{t-1}), \tag{26}$$

where (23) appears, therefore, the value of log-likelihood function is obtained through the forward phase. By using numerical optimization method such as quasi-Newton method, the optimal vector of parameters, denoted by $\hat{\mathbf{x}}$, can be obtained.

Minimum AIC [2, 3] procedure is used to determine the optimal order of the model and to select the optimal model. AIC for our model is defined as follows,

$$\mathrm{AIC} = -2l(\hat{\mathbf{x}}) + 2 \times (p+1). \tag{27}$$

In the minimum AIC procedure, the order that minimizes AIC is employed as the optimal order, and the model that has minimum value of AIC is selected as the optimal model.

3.4 Smoothing

After the order determination has been finished, smoothing process will be done. Since the smoothing process is going toward the opposite direction of time, this can be called backward phase of the estimation. Estimated result by backward phase is most reliable since all data are used for the estimation. There are several methods for smoothing, such as fixed point, fixed interval, and fixed lag methods. In this paper, we have employed fixed interval method as the smoothing procedure.

The exact distribution of smoothing of time t is calculated from smoothing and prediction distributions of time $t+1$, and filtering distribution of time t as follows,

$$p(\theta_t|Y_N) = p(\theta_t|Y_t) \int \frac{p(\theta_{t+1}|Y_N)p(\theta_{t+1}|\theta_t)}{p(\theta_{t+1}|Y_t)} d\theta_{t+1}. \tag{28}$$

The numerical implementation of smoothing becomes as follows,

$$s_i(t) = f_i(t) \sum_{j=0}^{M-1} \frac{s_j(t+1) \cdot q_{ij}}{p_j(t+1)} (b_{j+1} - b_j)^2. \tag{29}$$

4 Model Extension

We have employed non-Gaussian nonlinear state space modeling method [7, 8] for the estimation of the model. Since this method include numerical integration, dimension of state vector is limited by the computational cost. Practically, it is at most 4 by current computational resources. In our model, dimension of state vector corresponds to the number of peaks denoted by m.

In our model, there is a relationship between the number of spectral peaks m and the AR order p, denoted by $p = 2m$. Thus the AR order is limited by the computational reason mentioned above. This limitation is not suit for actual applications since low order might produces poor fitness and will estimate meaningless result.

To overcome this problem, we have extended the original model by introducing a stationary AR model of order n. Since the total order becomes $p = 2m + n$ by this extension, we have no limitation of AR order caused by computational cost.

The extended model is defined as follows. We construct multiplicative model between original TVPP model and the stationary AR model as follows,

$$\prod_{k=1}^{m} (1 - r_k e^{i\theta_k(t)} B)(1 - r_k e^{-i\theta_k(t)} B) \times (1 - \sum_{j=1}^{n} c_j B^j) = 0. \tag{30}$$

Where, coefficients c_j of stationary AR part are estimated by numerical optimization based on likelihood function. Then the vector of time invariant parameters (14) is modified by this extension as follows,

$$\mathbf{x} = \left[\sigma^2, \tau_1^2, \tau_2^2, \ldots, \tau_m^2, r_1, r_2, \ldots, r_m, c_1, c_2, \ldots, c_n\right]^T. \tag{31}$$

Note that the number of parameters is not changed to the original model with respect to the model order p. Thus the formula of AIC is the same of original model's (27).

Through the estimation of extended model, we obtain time varying peaks of power spectrum from $\mathbf{\Theta}_t$, and time invariant factor of power spectrum from $c_j, (j = 1, 2, \ldots, n)$. This is an advantageous feature of extended model since the spectral features of data are decomposed into time varying peaks and time invariant factor by one pass estimation.

5 Numerical Experiment

In order to show the efficiency of TVPP model, numerical experiments have been shown in this section. TVPP model has been applied to artificial data that have been generated by changing the peak frequencies of power spectrum. TVCAR model is also applied to the data, and the results of both models are compared together with respect to AIC, expected log-likelihood, and mean squares error of peak frequency.

5.1 Artificial Data

Three data sets, named data I, data II, and data III, are generated by following processes.

Process I (for data I):

$$\begin{aligned} \theta_t &= \theta_{t-1} + \delta\theta \\ y_t &= \sum_{j=1}^{2} a_j^{(2)}(\theta_t, r) y_{t-j} + \varepsilon_t \end{aligned} \tag{32}$$

($\delta\theta = 0.01$,$r = 1.2$)

Process II (for data II):

$$\begin{aligned} \mathbf{\Theta}_t &= \mathbf{\Theta}_{t-1} + \delta\mathbf{\Theta} \\ y_t &= \sum_{j=1}^{4} a_j^{(4)}(\mathbf{\Theta}_t, \mathbf{R}) y_{t-j} + \varepsilon_t \end{aligned} \tag{33}$$

($\mathbf{\Theta}_t = [\theta_1(t), \theta_2(t)]^T$, $\delta\mathbf{\Theta} = [0.004, -0.004]^T$, $\mathbf{R} = [1.2, 1.2]^T$)

Process III (for data III):

$$\begin{aligned} \theta_t &= \theta_{t-1} + \delta\theta \\ v_t &= \sum_{j=1}^{2} a_j^{(2)}(\theta_t, r) v_{t-j} + \varepsilon_t \\ y_t &= \sum_{j=1}^{2} c_j y_{t-j} + v_t, \end{aligned} \tag{34}$$

($\delta\theta = 0.004$, $r = 1.2$, $c_1 = 0.0$, $c_2 = -0.6944$)

For each process, pseudo random numbers are used as the values of $\varepsilon_t \sim N(0, \sigma^2)$ with $\sigma = 3.0$. The number of data is 300 in each data set.

5.2 Model Fitting and Estimation Result

Extended version of TVPP model and TVCAR model are applied to the data sets. The value of AIC obtained for several order of each model are shown in **Table 1** and **Table 2** for data I,

Table 3 and **Table 4** for data II, and **Table 5** and **Table 6** for data III. Since true processes are known in the experiments, the values of expected log-likelihood can be obtained and shown in these tables. The value of AIC and expected log-likelihood of optimal oreder are underlined in each table. The optimal order by minimum AIC procedure is agreeed with the order of true process, except data III of TVPP model. In this case, expected log-likelihood scores the same order as true process.

Estimation results are shown as follows. In TVPP model, peak frequencies of power spectrum can directly be obtained. Estimated peaks are shown in **Figure 1** for data I, **Figure 2** for data II, and **Figure 3** for data III. Since distributions of peak frequency are obtained by TVPP model, confidential interval of peak frequency can be displayed. In each figure, 50%(median), 68%, 95%, and 99% points are plotted by solid lines, and dashed lines show the true peaks. For data III, time invariant factor of power spectrum has been estimated, and it is shown in **Figure 4**.

In TVCAR model, time varying power spectrum is firstly obtained from estimated AR coefficient. Secondly, from the estimated spectrum, peak frequencies of power spectrum are numerically obtained. They are plotted in **Figure 5** for data I, **Figure 6** for data II, and **Figure 7** for data III. In these figures, solid lines show the estimated peaks, and dashed lines show the true peaks. Since time smoothness of peaks is not assumed in TVCAR model, peaks between adjoint time are discontinuously shown in each figure.

5.3 Discussion

Figure 1, **Figure 2**, and **Figure 3** show the estimated results by TVPP model for each data set. From the figures, smooth evolution of spectral peaks are easily recognized. Note that TVCAR model also estimates the evolution of spectral peaks as shown in **Figure 5**, **Figure 6**, and **Figure 7**, however, they contain several abrupt changes of frequency. Moreover, by looking at the result of data III shown in **Figure 7**, stable estimate of time invariant peak of power spectrum cannot be obtained by TVCAR model.

As shown in **Figure 1**, **Figure 2**, and **Figure 3**, confidential interval of time varying peak can be obtained by TVPP model. For all data sets, we can see that true peaks are covered by confidential interval of 95% in the most part of series.

Data III contains time invariant peak of power spectrum at angular frequency $\pi/2$. We can see that the decomposition of spectrum into time varying peaks of **Figure 3** and time invariant factor of **Figure 4** is available in TVPP model.

Comparison of estimation results between TVPP model and TVCAR model has been done based on AIC, expected log-likelihood, and mean squares error of peak frequencies. The values of these criteria are summarized in **Table 7**. By looking at the table, all criteria of all data sets show that TVPP model is much better than TVCAR model.

We can see that the order determination by AIC of TVPP model in case of data I and data II work correctly, and it does not in case of data III. All spectral peaks are varying with time in data I and data II, but not all in data III. Therefore we can say that order determination by AIC for TVPP model correctly works while all spectral peaks are varying with time. In data III, AIC still has bias compared with expected log-likelihood.

6 Concluding Remarks

A new model to estimate time varying peak frequencies of power spectrum has been proposed, and has been named TVPP model. The model contains nonlinear factor because of the definition of time varying peaks of power spectrum. In TVPP model, estimation method with the aid of non-Gaussian filtering method, and order determination and model selection based on AIC have been shown. The model has been extended to have time invariant factors of power spectrum.

The advantages of TVPP model are summarized as follows; 1) smooth evolution of power spectral peaks can directly be estimated, 2) confidential interval of time varying peak is obtained, and 3) decomposition of spectrum into time varying peaks and time invariant factors is available. These advantages are confirmed through numerical experiments. Also it is confirmed through the experiments that TVPP model can provide better result than TVCAR model with respect to AIC, expected log-likelihood, and mean squares error of peak frequency.

Future works of TVPP model are mentioned. There are many targets that have nonstationary spectrum such as seismic wave, vibration with nonstationary condition, and human voice. These are interesting applications of TVPP model. The advantages of TVPP model will be used in these applications as follows; smooth estimation of power spectral peaks will serve a new point of view in each application, confidential interval estimated by TVPP model will play an important role for decision making on a design or feature recognition of target, and decomposition of spectrum will be available to extract meaningful information from noisely features in actual applications.

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A Algorithm

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program Algorithm

  function ForwardPhase(m,x)
  begin
    t := p + 1;
    l(x) := 0;
    initialize prediction for t p(Θ_t|Y_{t-1}) := p(Θ_{p+1});
    while t ≤ N begin
      filtering for t p(Θ_t|Y_t);
      l(x) := l(x) + log p(y_t|Y_{t-1});
      if(t < N) prediction for t + 1 p(Θ_{t+1}|Y_t);
      t := t + 1;
    end;
    return l(x);
  end;

  function Optimization(m)
  begin
    foreach x = {x_1, x_2, ..., x_n} begin
      l(x) := ForwardPhase(m,x);
    end;
    let x* such as x* = max_x l(x) ;
    return x*;
  end;

  function OrderDetermination()
  begin
    for m = 1, 2, ..., m_max begin
      l*(m) := Optimization(m);
      AIC(m) := 2l*(m) - 2(2m + 1);
    end;
    let m* such as m* = min_m AIC(m);
    return m*;
  end;

  procedure BackwardPhase(m)
  begin
    t := N ;
    initialize smoothing for t p(Θ_t|Y_t) = p(Θ_N|Y_N);
    while t > p begin
      smoothing for t - 1 p(Θ_{t-1}|Y_N);
      t := t - 1;
    end;
  end;

begin
  m* := OrderDetermination();
  BackwardPhase( m* );
end;
```

List of Tables

Table 1: AIC and expected log-likelihood for simulation data I by TVPP model

	$(m=1)$		$(m=2)$	
n	AIC	$E\{\log f(Y)\}$	AIC	$E\{\log f(Y)\}$
0	1349.43	(-672.27)	1361.43	(-679.29)
1	1349.97	(-672.51)	1363.07	(-678.89)
2	1351.89	(-672.57)	1367.87	(-678.37)
3	1353.40	(-673.05)	1368.27	(-680.09)
4	1353.34	(-674.12)	1366.31	(-679.34)
5	1354.98	(-674.11)	1368.35	(-679.49)

Table 2: AIC and expected log-likelihood for simulation data I by TVCAR model

p	AIC	$E\{\log f(Y)\}$
2	1361.85	(-676.25)
3	1370.26	(-684.54)
4	1383.16	(-688.16)
5	1397.53	(-691.38)
6	1406.15	(-696.24)
7	1415.41	(-701.74)
8	1422.60	(-708.05)
9	1431.39	(-711.82)
10	1440.50	(-723.22)

Table 3: AIC and expected log-likelihood for simulation data II by TVPP model

	$(m=1)$		$(m=2)$	
n	AIC	$E\{\log f(Y)\}$	AIC	$E\{\log f(Y)\}$
0	1487.00	(-741.90)	<u>1362.28</u>	(-671.81)
1	1473.94	(-738.74)	1362.97	(<u>-671.57</u>)
2	1425.29	(-716.08)	1365.77	(-673.53)
3	1410.49	(-709.30)	1367.21	(-672.78)
4	1412.08	(-708.62)	1368.94	(-673.60)
5	1409.79	(-707.78)	1370.96	(-673.65)

Table 4: AIC and expected log-likelihood for simulation data II by TVCAR model

p	AIC	$E\{\log f(Y)\}$
2	1420.19	(-703.57)
3	1427.92	(-704.20)
4	<u>1381.90</u>	(<u>-677.68</u>)
5	1387.04	(-683.75)
6	1391.93	(-686.12)
7	1398.25	(-688.53)
8	1402.43	(-693.12)
9	1412.62	(-694.90)
10	1417.64	(-695.56)

Table 5: AIC and expected log-likelihood for simulation data III by TVPP model

n	$(m=1)$ AIC	$(m=1)$ $E\{\log f(Y)\}$	$(m=2)$ AIC	$(m=2)$ $E\{\log f(Y)\}$
0	1431.48	(-709.56)	1345.74	(-677.31)
1	1419.60	(-706.94)	1347.47	(-677.13)
2	1344.14	(*-669.12*)	1345.33	(-703.32)
3	1346.05	(-669.20)	1345.77	(-682.11)
4	*1340.68*	(-674.92)	1347.24	(-679.53)
5	1341.59	(-674.29)	1349.14	(-678.98)

Table 6: AIC and expected log-likelihood for simulation data III by TVCAR model

p	AIC	$E\{\log f(Y)\}$
2	1418.65	(-707.84)
3	1404.23	(-700.63)
4	*1363.96*	(*-677.86*)
5	1367.54	(-678.31)
6	1373.46	(-681.28)
7	1382.09	(-682.59)
8	1388.72	(-685.26)
9	1390.51	(-688.19)
10	1394.75	(-691.67)

Table 7: Comparison between TVPP model and TVCAR model

		TVPP model	TVCAR model
Data I	order	$m = 1, n = 0$	$p = 2$
	AIC	1349.43	1361.85
	$E\{\log f(Y)\}$	-672.27	-679.29
	MSE	0.01007	0.02816
Data II	order	$m = 2, n = 0$	$p = 4$
	AIC	1362.28	1381.90
	$E\{\log f(Y)\}$	-671.57	-677.68
	MSE	0.01393	0.13580
Data III	order	$m = 1, n = 4$	$p = 4$
	AIC	1340.68	1363.96
	$E\{\log f(Y)\}$	-669.12	-677.86
	MSE	0.007867	0.02826

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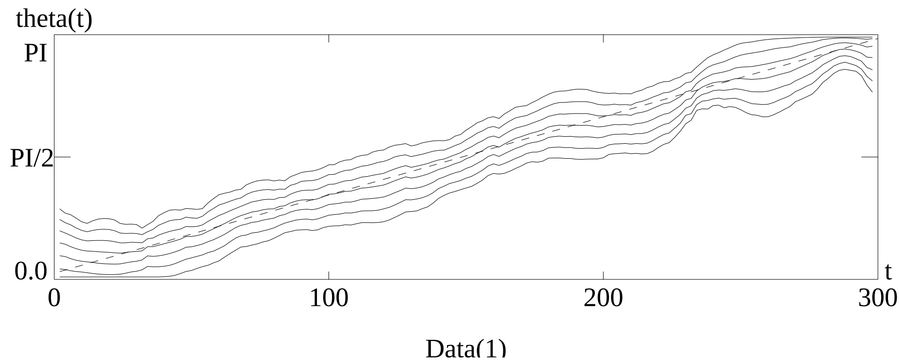

Figure 1: Estimated peak frequency of power spectrum by TVPP model for data I

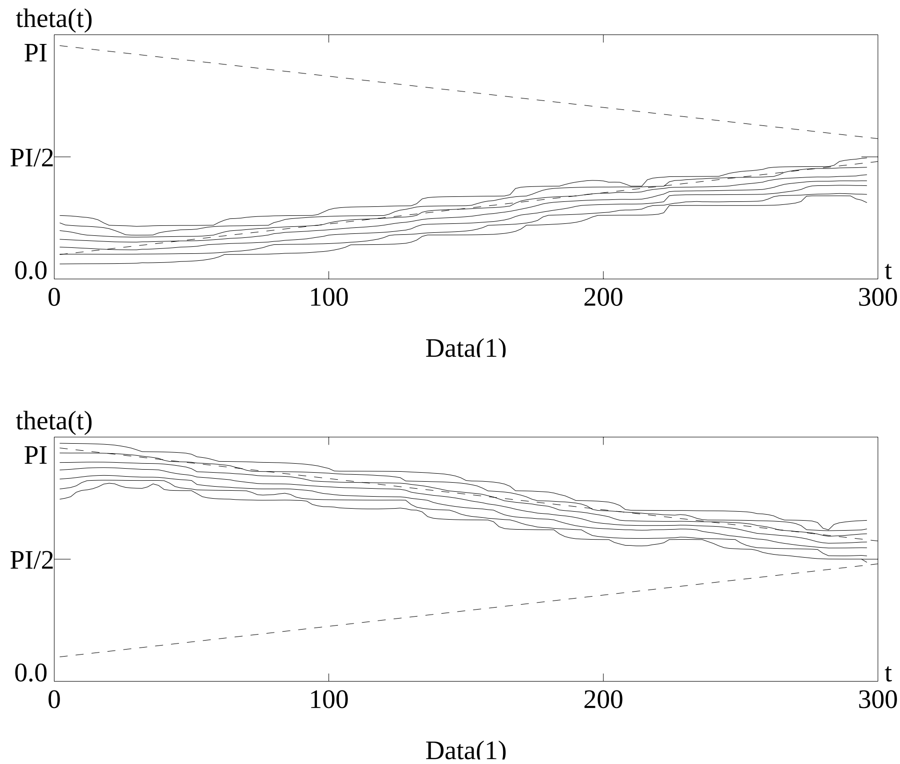

Figure 2: Estimated peak frequencies of power spectrum by TVPP model for data II

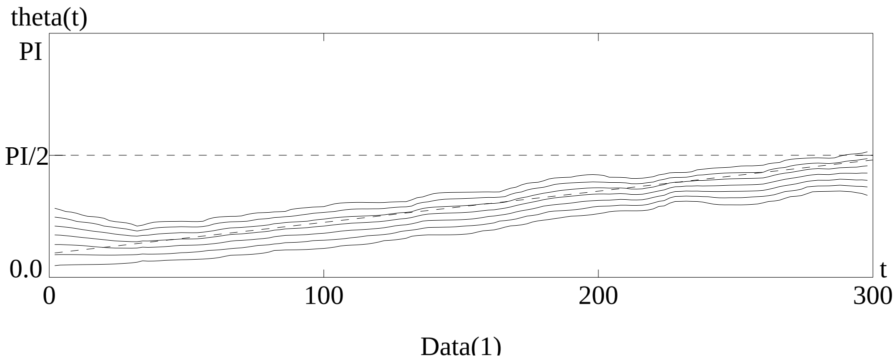

Figure 3: Estimated peak frequency of power spectrum by TVPP model for data III

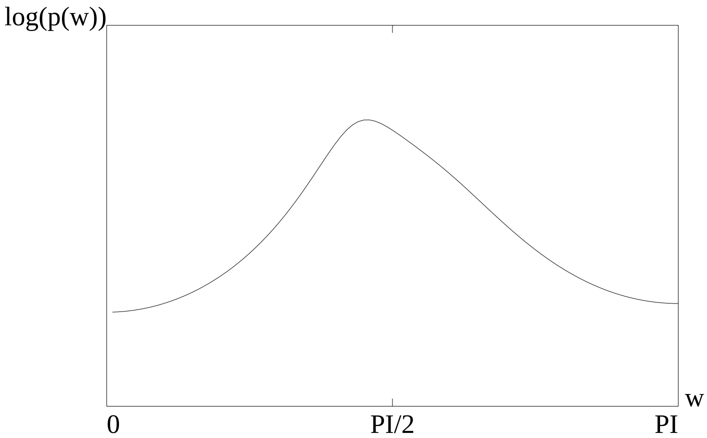

Figure 4: Time invariant power spectrum estimated by TVPP model for data III

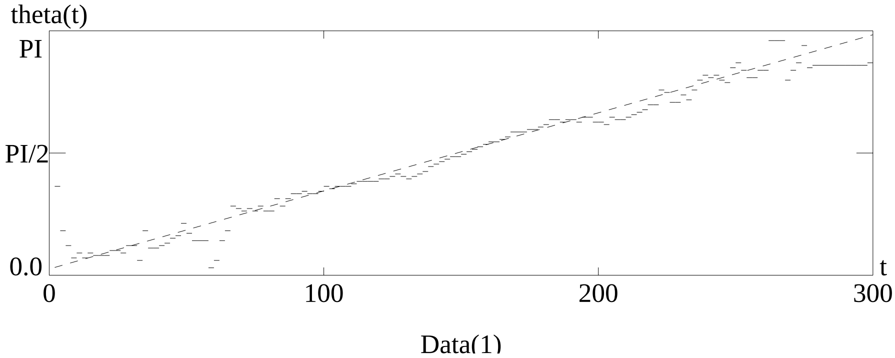

Figure 5: Estimated peak frequency by TVCAR model for data I

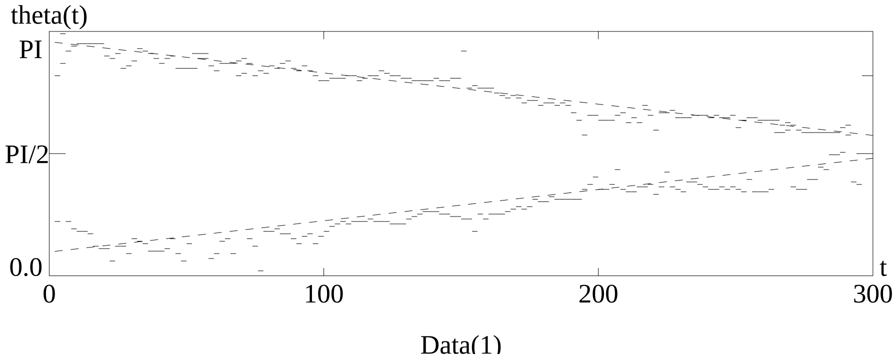

Figure 6: Estimated peak frequencies by TVCAR model for data II

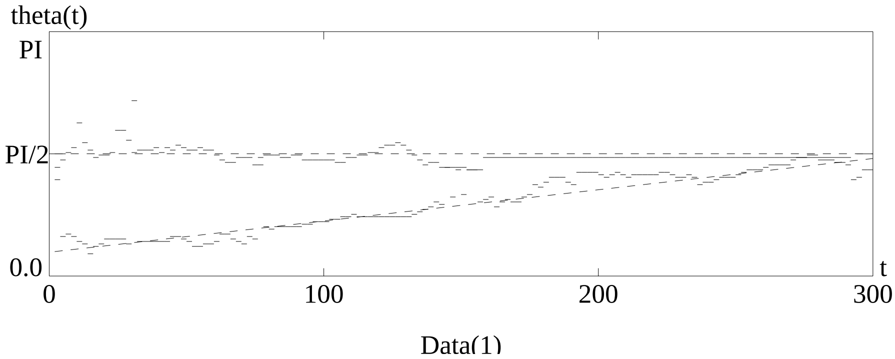

Figure 7: Estimated peak frequencies by TVCAR model for data III