Time Series Analysis based on Time-Varying Peak Frequencies of Power Spectrum and Application to Seismic Wave Data

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1.Introduction

A new model based on time-varying peak frequencies of power spectrum (TVPP model) has been proposed, and application result of it to seismic wave data has been reported. The original version of proposed model consists of system equation and observation equation. System equation has exponential part of characteristic roots of AR model smoothly changing with time, and observation equation is defined nonlinearly with respect to the state of system equation. Original model has been extended to improve fitness of the model to data by making multiplicative model between original model and stationary AR model. State estimation of the model while time invariant parameters are giving is done by nonlinear nonstationary state space modeling proposed by [2]. Time invariant parameters of the model is estimated by numerical optimization based on likelihood function. In application to seismic wave data, TVPP model has been examined, and also time-varying coefficient AR model(TVAR mode)[1] has been applied. Application results of both models are compared and discussed.

2.Time-Varying Peak of Power spectrum model(TVPP model)

Let vector of time-varying peak frequencies of power spectrum at time t be $\mathbf{\Theta}_t = [\theta_1(t), \theta_2(t), \ldots, \theta_m(t)]^T$, let us assume parameters related to power of each peak are time invariant and denoted by $\mathbf{R} = [r_1, r_2, \ldots, r_m]^T$. We define a model to estimate time-varying peak frequencies of power spectrum as follows,

$$\begin{aligned} \mathbf{\Theta}_t &= \mathbf{\Theta}_{t-1} + \mathbf{w}_t, \\ v_t &= \sum_{j=1}^{2m} a_j(\mathbf{\Theta}_t, \mathbf{R}) v_{t-j} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2), \\ y_t &= \sum_{j=1}^{n} c_j y_{t-j} + v_t, \end{aligned} \tag{1}$$

where, system noise $\mathbf{w}_t = [w_1(t), w_2(t), \ldots, w_m(t)]^T$ is a random vector accroding to normal distribution of 0 mean and Q covariance matrix ($Q = diag\left(\tau_1^2, \tau_2^2, \ldots, \tau_m^2\right)$). AR coefficient, $a_j(\mathbf{\Theta}_t, \mathbf{R})$, is derived by solving the following equations,

$$1 - \sum_{j=1}^{2m} a_j(\mathbf{\Theta}_t, \mathbf{R}) B^j = \prod_{k=1}^{m} (1 - r_k e^{i\theta_k(t)} B)(1 - r_k e^{-i\theta_k(t)} B). \tag{2}$$

Time constant parameters, σ^2,τ_1^2,τ_2^2,...,τ_m^2, r_1,r_2,...,r_m, c_1,c_2,...,c_n are obtained by numerical optimization to maximize likelihood function. Quasi-Newton method with BFGS modification formula of Hessian is used as the numerical optimization method.

State estimation of the model is done by nonlinear nonstationary state space modeling method proposed by [2]. This method can be divided into two phases, forward phase by applying filtering and prediction alternatively, and backward phase of smoothing. The value of likelihood which is used in numerical optimization is obtained in forward phase. After determined the optimal values of time invariant parameters, backward phase is done and we obtain smoothing estimation of state.

3.Analysis of Seismic Wave Data

Seismic wave data observed in Hokkaido, Japan has been shown in **fig.** 3. The data consist of east-west direction of vibration, and length of data is 2500 with sampling time 0.02 second. Initial vibration, primaly wave(P-wave), secondary wave(S-wave) and its decay are contained in the data. While the data has nonstationary variance, normalization of variance is required before applying TVPP model or TVAR model[1]. Variance normalized series of seismic wave data is shown in **fig.** 4. Values of AIC obtained by TVPP model and TVAR model for variance normalized series are shown in **tbl.** 1. TVAR model has better fitness than TVPP model in terms of AIC. Estimated result by TVPP model is as follows, frequency of time-varying peaks have been shown in **fig.** 5, and time invariant factor of power spectrum has been shown in **fig.** 1. Time-varying power spectrum obtained by TVAR model has been shown in **fig.** 2. Time-varying peak frequencies numerically obtained from the spectrum has been shown in **fig.** 6.

By looking at the estimated result of time-varying peaks by TVPP model, different frequencies are confirmed among initial vibration, P-wave, and S-wave. Also we can see that the frequency of S-wave is going to back to the original one as the amplitude of the wave going down.

References

[1] G.Kitagawa and W.Gersch: A Smoothness Priors Time-Varying AR Coefficient Modeling of Nonstationary Covariance Time Series, *IEEE Trans* AC-30, **1**, 48-56(1985).

[2] G.Kitagawa: Non-Gaussian State-Space Modeling of Nonstationary Time Series (with discussion), *Journal of the American Statistical Association* **82**,No.400,1032-1063(1987).

Table 1: Values of AIC for seismic wave data

n of TVPP model	TVPP model		TVAR model	
p of TVAR model	$(m=1)$	$(m=2)$	$(k=1)$	$(k=2)$
0	5532.13	4920.54	.	
1	5389.31	4896.59	6231.14	6249.99
2	5216.65	4893.50	5275.58	5382.85
3	4877.33	4737.70	4845.18	4890.47
4	4872.92	4729.95	4598.60	4608.77
5	4811.62	4731.02	4628.58	4634.89
6	5034.67	4671.28	4621.63	4616.72
7	4766.36	4636.29	4626.05	4618.25
8	4724.06	4614.22	4601.88	4604.13
9	4724.26	4612.76	4600.42	4618.90
10	4724.99	4629.40	4605.10	4644.37

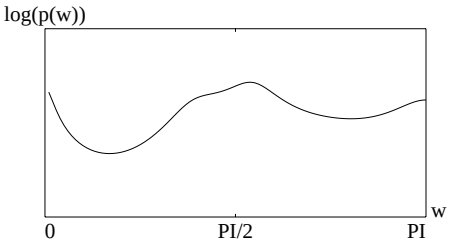

Figure 1: Estimated result of time invariant power spectrum by TVPP model

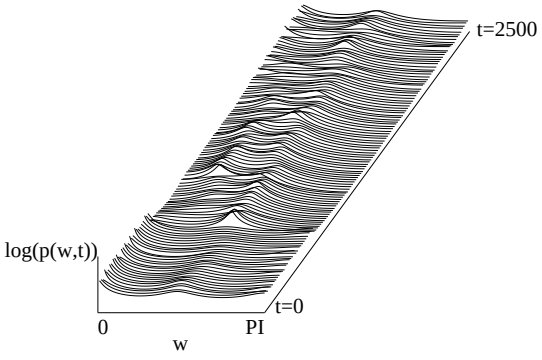

Figure 2: Estimated result of time-varying power spectrum by TVAR model

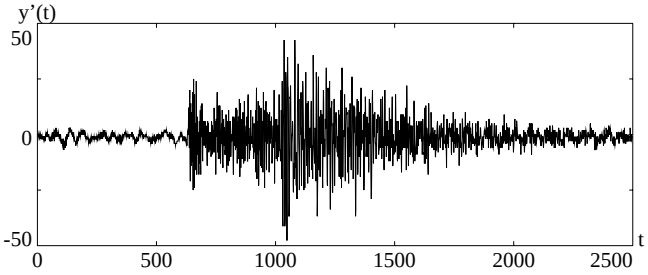

Figure 3: Seismic wave data

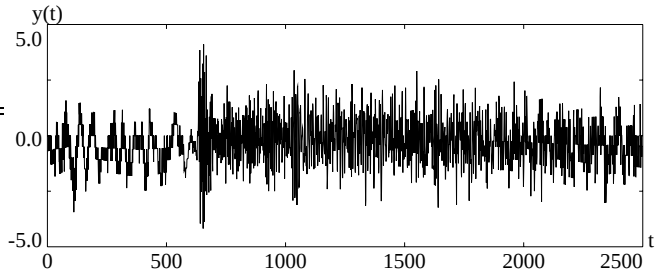

Figure 4: Variance normalized series of seismic wave data

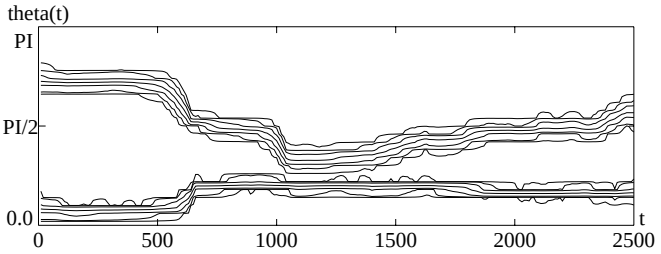

Figure 5: Estimated result of time-varying peak frequencies of power spectrum by TVPP model(50%,68%,95%,and 99% points are plotted)

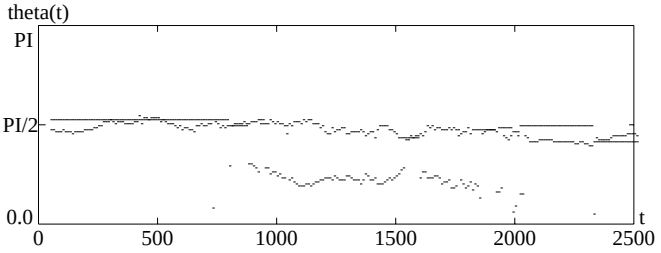

Figure 6: Estimate result of time-vaying peak frequencies of power spectrum by TVAR model